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BENJAMIN FRANKLIN AND THE POST OFFICE

BY

THOMAS COULSON[1]

The recent bequest of the Harry L. Jeffreys collection of postage stamps to The Franklin Institute serves to remind us of a somewhat neglected aspect of Franklin's diverse activities. His achievements in diplomacy, in politics, in science, and in literature are enshrouded in an aura of romance which has caused them to overshadow the more prosaic but not less useful work he did as the organizer of the American postal system. His thirty-seven years of zealous and often arduous administration did not permit the performance of any spectacular accomplishment, but the historian who neglects to study the influence of the improved methods of communication brought about by Franklin's labors will overlook one of the main currents which brought about the unification of the widely separated colonies.

However, it is doubtful that it was the hope of promoting a unity among the colonists which induced him to enter the postal service. Instead, it seems to have been a display of business acumen. The broader view of the field only came after he had gathered experience and the possibilities appeared in his mind. We shall allow these views to fit themselves into the appropriate chronological order as this brief history of the Post Office unfolds.

Needless to say, Franklin did not originate the postal system; the service was already old when he entered it. We shall gather an idea of its antiquity when we recall that the famous inscription on the New York General Post Office, "Neither snow, nor rain, nor heat, nor gloom of night stays these swift couriers from the swift completion of their appointed rounds," was written by the ancient Greek historian Herodotus in praise of the couriers of his day. Sundry forms of courier service were intro-

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(Note—The Franklin Institute is not responsible for the statements and opinions advanced by contributors in the JOURNAL.)

duced in several countries but it was not until 1512 that a postmaster was appointed in England to organize a system by which the royal correspondence might be transmitted and to provide post horses for the use of those who were engaged upon official business. From this, by a slow evolutionary process, the British Post Office has grown. Especially established to meet a governmental need, it passed into the hands of those who exploited it for their personal profit. When the acts of the exploiters became too monopolistic and their profits too exhorbitant, governmental intervention and regulation became inevitable. In the main, the American postal system followed the same pattern until in 1710 the Post Office Act of Queen Anne introduced reforms in both systems and the modern post offices were given their rudimentary forms.

Franklin's original association with the American Post Office sprang directly from his interests as the publisher of a newspaper. At the time no law regulated the distribution of newspapers through the mail. Consequently, it was a decided advantage to a publisher that he should receive appointment as a postmaster. Tenure of this office enabled him to gather news cheaply and expeditiously through the post riders and, best of all, it enabled him to distribute his newspaper free of cost. If he had the courage to do so, he might forbid his post riders to carry any rival journal. This was the situation that irked Franklin. He found his newspaper, the *Pennsylvania Gazette*, excluded from the mails by Andrew Bradford, the postmaster of Philadelphia, who was the publisher of the rival *American Weekly Mercury*.

However, the application of this monopolistic tactic had nothing to do with Bradford's expulsion from office and the appointment of Franklin in his place. Franklin was doubtless getting around the difficulty by bribing the riders to carry his newspaper, which was the accepted practice. The trouble was that Bradford had unorthodox views about the keeping of accounts and the rendering of financial statements. For three years in succession he had neglected to make any returns or payments. As he was supposed to render semi-annual reports, the Deputy Postmaster General for America must have exercised much forebearance before he made up his mind to dismiss the delinquent. Franklin records his appointment to the office in his "Autobiography" as follows:

> In October, 1737, Colonel Spotswood, late governor of Virginia, and then postmaster-general, being dissatisfied with the conduct of his deputy at Philadelphia, respecting some negligence in rendering and inexactitude of his accounts, took from him the commission and offered it to me. I accepted it readily, and found it of great advantage; for, though the salary was small, it facilitated the correspondence that improved my newspaper, increased the number demanded as well as the advertisements to be inserted, so that it came to afford me a considerable income.

The opportunity of pointing the moral was irresistible. Continuing on the subject of Bradford's dismissal, Franklin said: "Thus he suffered greatly from his neglect in due accounting; and I mention it as a lesson to those young men who may be employed in managing affairs for others, that they should always render accounts, and make remittances, with great clearness and punctuality. The character of observing such a conduct is the most powerful of all recommendations to new employments and increase of business." Although it is apparent that Franklin's main profit from the postmastership came from the prosperity it brought his newspaper, he was not unmindful of his official duties. The following advertisement appeared in the *Gazette* of October 27, 1737:

> Notice is hereby given that the post office of Philadelphia is now kept at B. Franklin's, in Market Street; and that Henry Pratt is appointed Riding Postmaster for all the stages between Philadelphia and Newport in Virginia, who sets out about the beginning of each month, and returns in twenty-four days; by whom gentlemen, merchants, and others, may have their letters carefully conveyed, and business faithfully transacted, he having given good security for the same to the Honorable Colonel Spotswood, Postmaster-General of all His Majesty's Dominions in North America.

One would naturally suppose that Franklin, after his admonition upon the failure of Bradford, would discharge his duties diligently and that his accounts were scrupulously accurate and punctual. The conscientious manner in which his work was done induced the Postmaster General to appoint him Comptroller, a position which ranked as next in importance to that of the Deputy himself, and which gave him an intimate knowledge of the internal economy of the system that was to prove exceedingly valuable at a later date.

There is no reason to doubt that he found the position as postmaster a congenial occupation, apart from its effect upon his newspaper's circulation. Franklin enjoyed the companionship of his fellow men and it was his contact with people in every walk of life which gave depth and expanse to his philosophy. His position as postmaster greatly enlarged his acquaintances, for the delivery of the post was something of a social occasion, the significance of which has escaped the brush of the artist and the pen of the historian. Especially at times of crisis, such as the war in 1748, the arrival of the mail was the signal for the gathering of a crowd of anxious inquirers at his shop where, if they did not receive letters themselves, they could at least gather the latest news from those who were more fortunate. Franklin the editor was, therefore, at the fount of public opinion at such times.

Meanwhile, he prospered to such an extent that he was able to retire from active participation in his business as a printer, and began to exert himself more fully in public affairs. His retention of the Post Office would indicate that he regarded it less as a source of income than as a public service. He was to hold many offices but he sought appointment to very few. Yet, having acquired a perception of the value the Post Office could come to have as the unifying agent for the Colonies, he deliberately sought appointment to the office of Deputy Postmaster General for America. The incumbent, Elliot Benger, was in ill-health and little hope was given for his recovery, whereon Franklin began to pull strings in order that he might secure the reversion of the office. To this end he wrote to his correspondent in London, Peter Collinson, soliciting his aid.

> . . . The occasion of my writing this, via Ireland is That I have just receiv'd advice that the Deputy Post Master General of America (Mr. Elliot Benger, residing in Virginia) who has for some time been in declining Way is tho't to be near his End. My friends advise me to apply for this Post and Mr. Allen (our Chief Justice) has wrote the enclos'd to his Correspondent, Mr. Simpson, in my favor requesting his Interest and Application in the Affair and impowering him to advance a considerable Sum if it should be necessary.
>
> I have not hithertofore made much scruple of giving you Trouble when the Publick Good was to be promoted by it, but 'tis with great Reluctance that I think of asking you to interest yourself in my private Concerns, as I know you have little time to spare. The Place is in the Disposal of the Post Masters General of Britain with some of whom or their Friends you may possibly have Acquaintance. Mr. Allen has desir'd Mr. Simpson to confer with you on the Affair and if you can without much Inconvenience to yourself advise and assist in endeavouring to secure the Success of this Application you will, whatever may be the Event, add greatly to the Obligations you have already conferr'd on me: and if it succeeds I hope that as my Power of doing good increases my Inclinations will always at least keep pace with it. I am quite a Stranger to the Manner of Managing these Applications so can offer no particular Instructions. I enclose a Copy of the Commission of a former Deputy Post Master General which may be of some use. The Articles of Agreem^{t} referr'd to in the Commission I have never seen but suppose they have always been nearly the same whoever is appointed, and have usually been sent over to America to be executed by the new Officer; for I know neither of the three last Officers went to England for the Commission. The Place has commonly been reputed worth about 150 pounds a Year but would be otherways very suitable to me, particularly as it would enable me to execute a Scheme long since form'd of which I send you enclos'd a Copy, and which I hope would soon produce something agreeable to you and to all Lovers of Useful Knowledge, for I have now a large Acquaintance among the ingenious Men in America. I need not tell you that Philadelphia being the Center of the Continent

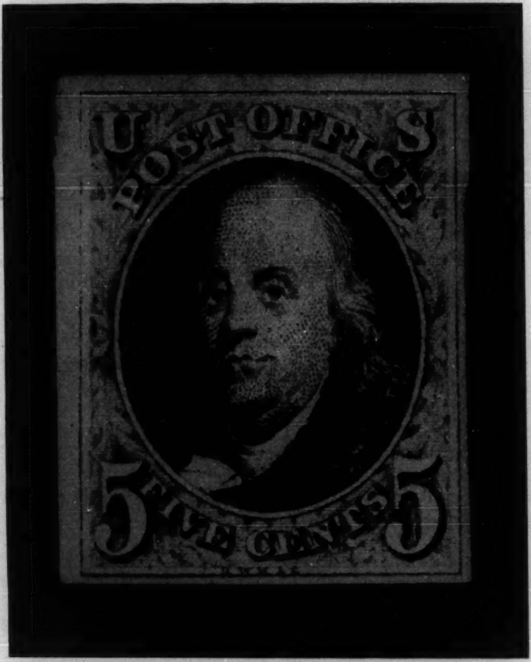
> Colonies and having constant Communication with the West India Islands is by much a fitter Place for the Situation of a General Post Office than Virginia, and that it would be some Reputation for our Province to have it establish'd here. I would only add that as I have a respect for Mr. Benger I should be glad the Application were so managed as not to give him any Offense if he should recover. But I leave everything to you and Mr. Simpson referring you to Mr. Allen's Letter to that Gentleman for further particulars, and am dear Sir,
>
> Your affectionate humble Serv^{t}
> B. Franklin
>
> P.S. I have heard 200 pounds was given for this Office by Mr. Benger and the same by his Predecessor. I know not whose Perquisite it was. But lest that not be sufficient and there may be some contingent Fees and Charges Mr. Allen has ordered 300 pounds. However the less it costs the better as 'tis an Office for Life only which is a very uncertain tenure.

However, the solicitor was somewhat premature in his application for Benger refused to be accommodating. He clung to life and office and did not leave his post vacant until two years later. Finally, on August 10, 1753, the following order was issued:

> Ordered that Mr. Benjamin Franklin, of Philadelphia in Pennsylvania, and Mr. William Hunter of Williamsburg in Virginia, be appointed Deputy Post Masters and Managers of all His Majesty's Provinces and Dominions on the Continent of North America, in the stead of Elliot Benger, Esq., Deceased, to commence this day, at an Allowance or Salary of 600 pounds p. Ann. to be paid out of the money arising from the Postage of Letters passing or repassing throughout the said Provinces and Dominions of North America.

The joint deputy, Colonel William Hunter, was publisher of the *Virginia Gazette*, but he was in such poor health that he was not able to play an active part in the administration of the postal system at times. However, upon his appointment Franklin immediately visited him and the pair made a tour of inspection of the southern Post Offices. At first glance it might appear that the office of Deputy Postmaster General would be a lucrative post, for the salary of three hundred pounds was a respectable sum in those days when money had a much larger purchasing power, but it should be observed that the salary was to be paid out of the profits of the postal system and, as the system had never, as yet, managed to cover the cost of its maintenance, the hopes of a salary to be derived from it were somewhat visionary. However, Franklin had ideas for improving the service and he hoped by applying the principles of good management to make it not only self-supporting but profitable enough to pay the salaries. But in the meantime, Franklin and Hunter discovered that they had become speculators, for they had to dig into their own purses to pay the expenses of the innovations they adopted. Within the period of four years the Post Office owed them four hundred pounds.

Franklin was too sagacious to permit the office to become a drain upon his resources without receiving some compensation. He appointed members of his own family to salaried posts in a manner which would provoke howls of protest today. His son William succeeded him as postmaster of Philadelphia, and later Comptroller. When William took over the latter office he was succeeded in the postmastership by Joseph Reed, a relative of Mrs. Franklin, and was followed in the course of time by Peter Franklin, who was brought from Boston to fill the post. The Post Office in Boston was given to Franklin's brother John, and when he died to his widow, who thereby became the first postmistress in America.



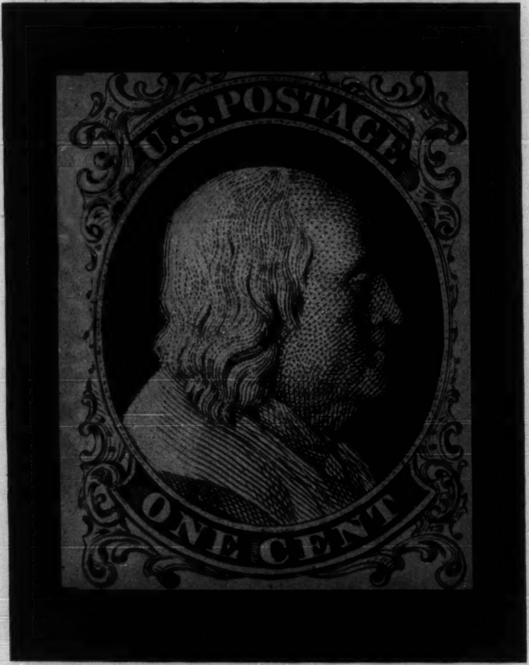
First adhesive postage stamp, 1847. Portrait of Franklin from a drawing by James B. Longacre.

However, it must not be supposed that Franklin was activated by selfishness in assuming the office of Deputy Postmaster General. It is characteristic of the man that he should take a large view of his duties and that his influence in their performance should extend far beyond the domain of the Post Office. His letter to Peter Collinson had indicated he had a wider view in prospect than the mere reform of a bankrupt system. His purpose became clearer as time passed.

He attended the Albany Congress in 1754 as a delegate from Pennsylvania and offered for discussion a plan which would draw the colonies into unified action, but the proposal only served to throw into sharper focus the variety and diversity of provincial prejudices and antipathies hindering a community of feeling. As Franklin acidulously stated: "Everybody cries a Union is absolutely Necessary, but when they come to the Manner and Form of the Union, their weak Noddles are perfectly distracted." There was nothing to be done at the moment but to accept the want of decision on the part of the other delegates, but it is a tenable supposition that a man of Franklin's political acuity should perceive not only the defects in the situation but also the means of providing a remedy. In brief, his remedy was an elaboration of the scheme hinted at in his letter to Peter Collinson. A more efficient postal system could be the means of drawing the Colonies into a better understanding of their common interests and of knitting them into a homogeneous community. The breaking down of the separating barriers of thought was the only way in which the widely scattered colonists could be diverted from concentration upon local problems into thinking upon the major problems that affected them as a national entity.

Franklin was aware of the immensity of the problem confronting him. After he had founded the American Philosophical Society he had reason to deplore the lethargy of its members, but a little reflection must have convinced him that the absence of roads and the primitive means of communicating intelligence were grave handicaps for the members to overcome. Living in an age when the transmission of news is almost instantaneous, it is difficult for us to visualize existence in a community where the latest news was invariably stale. There were no daily newspapers, and those which were published weekly were often content enough to publish information of events which had taken place during the previous month. Letters were scanty in number because of the expense of their transmission. The scientists of the day, or philosophers as they chose to call themselves, to whom knowledge of what others were doing in their field was almost as necessary as the air they breathed, were always lamenting the delays in the diffusion of knowledge. To Franklin, with his questing mind, the need for quicker and cheaper transmission of intelligence, not only of a scientific nature but in every form of human activity, was imperative if the Colonies were to accept a recognition of their need for unity.

The letter to Collinson, in which he wrote that his appointment to the Postmastership "would enable me to execute a scheme long since form'd . . . and which I hope would soon produce something agreeable to you and to all Lovers of Useful Knowledge," undoubtedly had reference to his desire to quicken the correspondence among the "ingenious Men in America" united in the American Philosophical Society. The necessity for an enlargement of his plan and its more pressing urgency became apparent when he had to deal with an embryo nation. The manner in which the postal service could be applied would spring from his experience as Comptroller.



One cent stamp, 1851. Profile, probably after Houdon's bust of Franklin.

The postal system which Franklin and Hunter inherited in 1753 was still under the establishment of the Postal Act of 1710. This was the great charter of the British Post Office until the reforms introduced by Rowland Hill in 1840. It was of itself a reform act but like so many statutes of that nature its defects were as conspicuous as its good intentions. The Act consolidated the posts of the entire British empire, and fixed the rates which could be charged. New York was intended to be the center of the system in North America (although it never did become so until many years later) and all rates were reckoned from that city with a fine disregard for the distances to the places named. To send a letter to the West Indies or to a point within 60 miles of New York cost four pence; to Perth Amboy or a point not exceeding 100 miles distant, six pence; to Philadelphia or New London, nine pence; to Boston or Annapolis, one shilling. The general rates from these places to way Post Offices was on the basis of four pence for 60 miles or six pence for 100 miles.

High prices were not the only deterrent to frequent use of the mails. The time consumed in transmitting letters was appallingly slow according to modern standards. Some effort was made to arrive at speed between the larger towns where roads existed, but even here the riders could encounter exasperating delays. It is needless to point out that the Colonies had developed along the Eastern shores, so that the post riders had to cross numerous unbridged rivers and creeks at their widest points. The Post Office Act instructed ferrymen to give the riders free and speedy passage, but what ferryman was going to row his cumbersome boat across the river and back without recompense for his exertions? The post riders were most often compelled to kick their heels in idleness until a paying load appeared, and this might mean a delay of hours.

This, be it remembered, was upon the better travelled highways. Along the roads that were less frequently used the speed was even slower. Here the post riders were frequently gray haired men whose days of activity were passed. They were content to travel at a more leisurely pace, dropping the reins on their horses' necks so that their hands might be free to knit socks or mittens as the animals ambled gently over the track.

In general, the worst feature of the Act was the toleration afforded private carriers of the post. They, and it must be confessed some of the official postmasters, allowed their riders to transport merchandise, carry money to pay bills, drive oxen, and otherwise display a delightful versatility for earning profit at the expense of the official post. They were willing to undertake any errand if only the delay to the mails it caused could be attributed to mischance, bad weather, or impassable roads.

This was the unsatisfactory situation which confronted Franklin and Hunter when they assumed office. Doubtless they would have gladly instituted reforms to put the post offices upon a self-supporting basis without delay, but Franklin had no sooner returned from Albany than he was compelled to give his attention to the Braddock expedition. This employment was not undertaken voluntarily. It was thrust upon him as the Deputy Postmaster General. His "Autobiography" makes this point clear.

> Our Assembly apprehending from some information that he [Braddock] had conceived violent prejudices against them, as averse to the service, wish'd me to wait upon him, not as for them, but as postmaster-general, under the guise of proposing to settle with him the mode of conducting with most celerity and certainty the dispatches between him and the governors of the several provinces.

The outcome of this was that Franklin had to undertake a personal survey of the roads to establish communication between Philadelphia and Braddock's camp in Virginia, and to provide the transport wagons the General demanded. The journey was to give rise to an incident which was to cause Franklin no little trouble. He relates in his "Autobiography" that when he left Philadelphia he was accompanied by thirty or forty uniformed troopers of his regiment, who escorted him to the Lower Ferry. This was reported to the Proprietor who, offended by Franklin's usurpation of what he regarded as his privilege, demanded that the Postmaster-General be dismissed from office.

We see his solicitude for promoting better social intercourse among the outlying districts in his efforts to develop the initial impulse given to one territory by his survey of the route to Winchester for General Braddock. This survey was primarily of a military nature, but the post rider who travelled over it once a week could carry private letters to Lancaster, York, and the settlements in the Cumberland Valley. Franklin knew that a mail service over this route could not be self-sustaining, yet he urged the Assemblies to improve the road, well knowing that improvements in communication would result in the development of the country through which the road passed. He clutched at every chance of fostering inter-colonial projects.

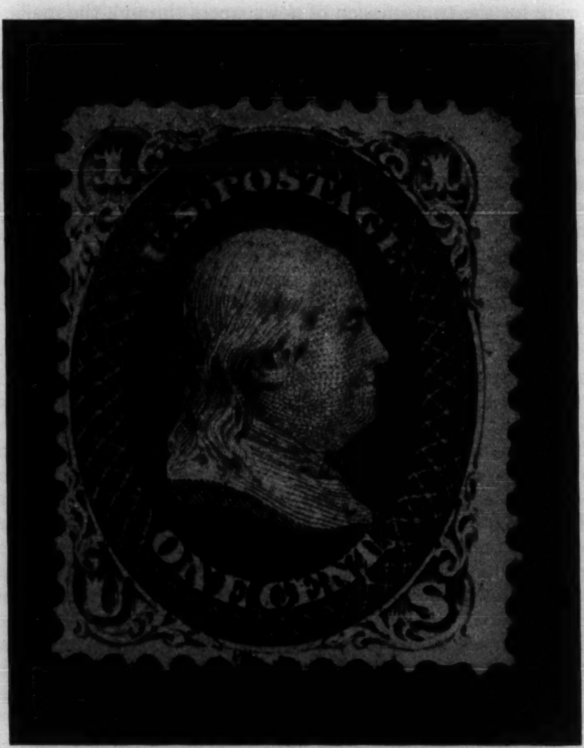
Meanwhile he was improving the mail service. There was not yet any attempt at delivering letters directly to the persons to whom they were addressed. Letters were delivered to the Post Offices along the post roads and remained there until called for. While postmaster of Philadelphia, Franklin had developed a means of facilitating delivery within the city and its immediate surrounding area by publishing in his *Gazette* the names of those for whom letters were being held in the post office. This example was quickly adopted in Boston, so that friends and acquaintances in both cities assisted in the delivery of the mail. The issue of the *Gazette* for July 26, 1753, contained the announcement of the first city delivery.

> Whatever Letters for Persons living in Town, remain uncall'd for, on those days they are brought to the Post-Office, will the next morning be sent out by a Penny Post provided for that purpose.

While this was an innovation on the American scene, the practice had already been in operation in England, where it had been tried in

London toward the end of the seventeenth century. It had proved to be such a boon that Franklin readily adopted it. He was soon employing three carriers to deliver his penny post.

Two years later he established the Dead Letter Office by instructing the postmasters that all letters remaining uncalled for at the end of three months were to be forwarded by the local postmasters to the central post office in Philadelphia.



One cent stamp, 1861. Profile after Houdon's bust.

One of Franklin's acts of magnanimity rebounded upon him. Aware from his own experience as a victim of the injustice of the actions of those postmasters who were also newspaper publishers and who excluded competing journals from the mails, he had, as soon as he was appointed a local postmaster, undertaken to accept for delivery all newspapers free of charge. This is additional evidence of his solicitude for the diffusion of information. Before many years had passed the number of journals began to increase to such an extent that their conveyance was burdensome to the carriers. The Deputy Postmasters had no authority to lower the rates, which would have done so much to increase the volume of business, but they had already spent so much out of their own pockets in maintaining the Post Office, that they now decided to impose a uniform rate for the transmission of newspapers through the mails.

No one will be surprised that the orderly and methodical mind of Franklin rebelled at the negligent manner in which the accounts were kept by local postmasters. He promptly instituted a more rigorous method of accounting by which one postmaster's accounts could be checked against another's. Still, this did not eliminate all the opportunities for the dishonest to profit, but beyond doing what he could to engage trustworthy men, Franklin could do very little to check the theft of money paid on letters gathered on the way and accepted for delivery by the riders. Nevertheless, superior management began to have its effect. For seven years Franklin and Hunter continued to cover the deficits of the Post Office from their own pockets. They did this hopefully, because each year they saw the revenues increase and they, naturally, entertained the hope that the receipts would sooner or later exceed the expenses. But it was seven years before the pair arrived at the point where they could recompense themselves and pay to the General Post Office in London the first profits to be derived from the American Post Office.

The revisions they had made in the postal service were sufficient to carry it forward while both Franklin and Hunter were out of the country. The latter had gone to England for reasons of health and, in 1757, Franklin had been employed by the State of Pennsylvania as their agent in London. During their absence the Deputies were represented by the Comptroller, James Parker, of New York, who had been a business partner of Franklin in the printing trade. Parker was a level headed business man and, if not much of a leader, was able to give a creditable account of himself in the direction of the Post Office. The correspondence conducted between them enabled Franklin to keep in close touch with affairs, and to give Parker a good deal of advice upon the management of the office.

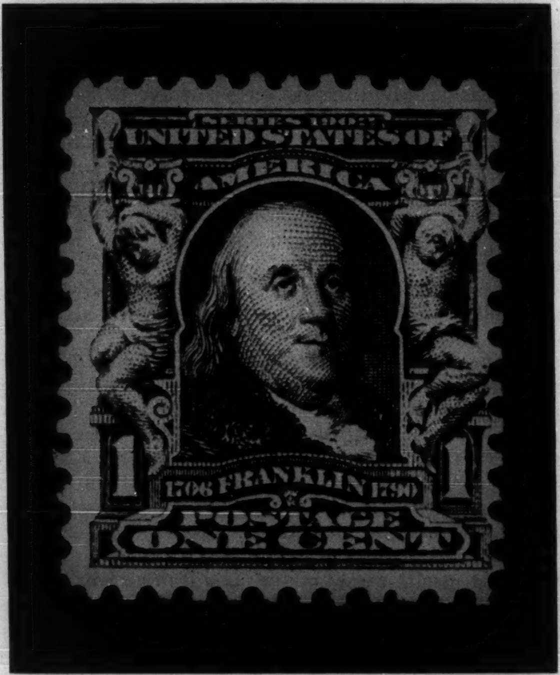
His absence in England lasted until 1762, and when he returned home it was under a new commission. His former partner in office, Hunter, had finally succumbed to his ills, and his place had been taken by John Foxcroft, another Virginian. The next few years were a very busy period in Franklin's public life but his devotion to the duties of the Post Office was unstinted. In the spring of 1763 he set out upon another of his extensive surveys of roads and inspection of the local post offices. Not that there were so many offices, but the seventy-five in operation were scattered all the way from Maine to Virginia. South of the latter point practically all mail was carried by sea. On this particular journey Franklin drove a light two-wheeled vehicle, and was accompanied by his daughter Sally, who usually rode on horse-back. Altogether, he covered nearly 1600 miles on this journey.

We have made more than one allusion to Franklin's activities in the development of roads in the Colonies to promote intercourse and trade, and it was probably upon this occasion that he took another step in this direction. Certainly it was upon one of these journeys that he marked the distances by milestones. He had placed on the axle of his carriage an odometer which is preserved in The Franklin Institute Museum. This instrument measured the distances he travelled and at every mile covered along the road from Philadelphia to Boston he caused a stone to be erected on which was cut the distance between one important place and another. These stones were also plentiful in the neighborhood of Philadelphia, being found on the Lancaster, York, Haverford, Gulph, Ridge, and Bristol roads.

There are reasons why we should think this journey was the occasion on which these stones were erected on the Boston Post Road. After he had completed the journey Franklin was able to give a very close measurement in miles of the distance he had covered, the only time when he was so detailed in the measurement of a journey. Another reason is to be found in the innovations which were soon introduced into the postal system. Franklin had been wooing and cajoling the British authorities in order to secure a reduction in the postal rates. The purchasing power of money was so relatively high in those days as compared to ours, that the rates discouraged the use of the mails. The post was a luxury too few people could afford. Franklin had long been convinced that lowering the rates would result in an increase of bulk in the correspondence carried and would increase the revenue.

Of greater significance than the prosperity of the Post Office, which was desirable of itself, was Franklin's political motive. The Colonies were separated by barriers of almost impenetrable wilderness and great distances. Franklin seemed to stand alone in his perception of the means by which these obstacles might be overcome. He was intent upon drawing the Colonies together, upon developing a common understanding, and upon speeding the conviction that future welfare of the respective communities must all follow the same course. He held tenaciously to the view that a cheap postage and dependable service would speed the unification of the American people.

He dared not give public expression to his idea so long as the colonists were agitating for release from the control of British overlords, whose strength resided in the divisions among the colonists, but he worked indefatigably to improve the service so that his object might be achieved. Unconscious that they were aiding Franklin in his major scheme, those colonists who had only material gains in view were giving him support. The colonists were loud in their protests over the cost of carrying correspondence, and it must have been evident to everyone acquainted with the situation that a well-filled carrier's bag on the New York to Philadelphia road must have defrayed the cost of carriage several times over. The only argument which could be advanced against this practice was that the surplus gained on the popular roads helped to maintain service on the less popular. The general abuse of the franking privilege only served to inflame the denunciations of those who complained about the expense of using the official mails.



One cent stamp, 1902. From the Duplessis portrait of Franklin.

In spite of reluctance upon the part of the British authorities to grant the American Postmaster's request for a reduction of rates, the postal service was being steadily improved. Several years earlier, Franklin had proposed to run "a stage wagon" once a week to Boston for the purpose of carrying mail, but his more conservative contemporaries had shaken their heads in disapproval at such a radical suggestion to conquer space and time. The usual method of carrying mail was on horseback, but not at all in the strenuous manner of the later "Pony Express." The riders on the less prosperous routes were disinclined to maintain regular schedules. They preferred to wait until a paying load of correspondence had accumulated. Franklin struggled energetically against this feeling of inertia. He was continually urging greater speed and more frequent trips. He increased the number of riders on the northern roads which were under his immediate control. As the outcome of his inspection in 1763 he revised some of the routes so that the riders might save time. Not all his innovations met with universal acceptance.

There were, for example, five ferries to be used on the road between New York and Philadelphia. A new route was devised to reduce this number to three, thereby reducing the time wasted by the riders in waiting for paying passengers to arrive. But the road had been diverted from passing through Perth Amboy, where the Governor of New Jersey was in residence. Unfortunately, his British superiors reproved the Governor for neglecting his correspondence at the time, and the reproof had greater force since the same authorities had been at much pains to increase the packet service between Britain and America. The Governor eagerly grasped at the change in the post road as an excuse for the delay in his letters. Franklin handled the excuse expertly when it reached him. He pointed out that if the Governor would send his servant one mile away to meet the post rider he could save much time in the collection and delivery of mail, which was more frequent than it had been when the road lay through Perth Amboy. Just how much the general situation had changed through the innovations and improvements is shown in a letter written by Franklin to Anthony Todd, the secretary to the General Post Office, in 1764.

> I will now just mention that we hope in the spring to expedite the communications between Boston and New York as we have already that between New York and Philadelphia by making the mails travel by night as well as by day, which has never hitherto been done in America. It passes now between Philadelphia and New York so quick that a letter can be sent from one place to another, and an answer received the following day, which before took a week, and when our plan is executed between Boston and New York letters may be sent and answers received in four days, which before took a fortnight, and between Philadelphia and Boston in six days, which before required three weeks. We think this expeditious communication will greatly increase the number of letters from Philadelphia and Boston by the packets to Britain.

The improvement in the service and the increase of revenue derived from it had raised the opinion of Franklin held by the British authorities. They finally came around to his economic theory that a decrease in rates would so encourage the use of the mail that the bulk carried would increase rather than diminish the revenue. In 1765, an Act was passed to lower the domestic rates generally 30 per cent. The franking privilege was largely withdrawn and some minor provisions for regulating the carrying of the mails by ship captains who collected from and delivered to coffee houses were incorporated in the Act. This would have constituted a triumph for Franklin had the results of his wisdom and perseverance not been almost entirely obliterated by the passing of the Stamp Act in the following year.

Because the new rates, like the old, were based upon the mileage over which the letters were carried, more stringent measures had to be adopted in the calculation of the distances. The new Act required that the surveyors of the roads had to record the distances under oath. It was this new provision, prompted by the reduction of rates, which doubtless prompted Franklin to mark the Boston post road with milestones.

In September, 1765, a new commission was issued to Franklin as Deputy Postmaster General because of the enlargement of the postal service to include Canada. Franklin and Foxcroft were confirmed in their offices with jurisdiction over the entire territory between Canada and Virginia. The Southern offices in North and South Carolina, Georgia, Florida, and the Bahama Islands, where the post was almost entirely water-borne, were created as a separate district under Benajmin Barons.

Franklin returned to England a few months before the passage of the Stamp Act. Naturally, he was strongly opposed to a measure which was so obnoxious to the colonists whom he represented. He was examined before the House of Commons when repeal of the Act was under consideration, and the position of the Post Office was the subject for much of his testimony. The astute diplomat arranged with a friend that the direct examination should be designed to enable him to offer evidence which would prove the impracticability of distributing the stamps through the Post Office, a task which he looked upon with the greatest disfavor. Later, when the supporters of the Stamp Act questioned him, Franklin refuted the suggestion that money paid for postage was a form of taxation. He stoutly maintained it was payment for a service rendered by the Post Office.

During Franklin's absence in England the Post Office underwent no change. Foxcroft continued to administer it (much to Mrs. Franklin's annoyance) from the Franklin home, where the headquarters remained despite the official designation of New York as the center of postal affairs. Parker continued as Comptroller and he maintained a steady flow of correspondence with Franklin. The burden of his letters had to do with complaints of his own misfortunes and the deterioration of the postal service. Franklin did what he could to remedy Parker's troubles, but he was powerless to amend the situation which arose from the rapidly growing reluctance of the people to have anything to do with the agencies of the British administration, of which the Post Office was one.

In the midst of a general dissatisfaction the post riders began to take advantage of their opportunities. There were widespread reports of corruption in the revival of the practice of carrying letters for personal profit. All manner of collusion was developed between the riders and business men to defeat paying the Post Office for the transmission of mail. Indeed, the riders were becoming so independent that they neglected to blow their post horns so that people who had letters for collection by the official channel might be warned of the rider's approach, and they discarded the locked portmanteaux with which they were provided, in favor of open sacks into which they could place letters or withdraw them without any accounting having to be rendered. Common carriers made their re-appearance as carriers of mail. Postmasters became careless in making returns and settling their accounts. It was evident that the controlling hand no longer existed.

Reports of the deficiencies in the service descended upon Franklin in a steady stream. He was profuse with advice, recommending that the negligent postmasters should be dragged before the courts. But he was having trouble of another sort. After his examination on the effects of the Stamp Act it became clear that he must clash with all those who were in favor of drastic action against the recalcitrant colonists. When Lord Sandwich, who supported the maintenance of the Stamp Act, was appointed Postmaster General it became evident that Franklin's tenure of office was in jeopardy. Those who argued that he should be relieved because his absence prevented him from fulfilling his duties to the Post Office were reinforced by malicious friends of the Penn family who were incensed by Franklin's efforts to reduce the proprietor's powers in Pennsylvania. Thus Franklin became the center of an intrigue to have him dislodged as a Crown servant.

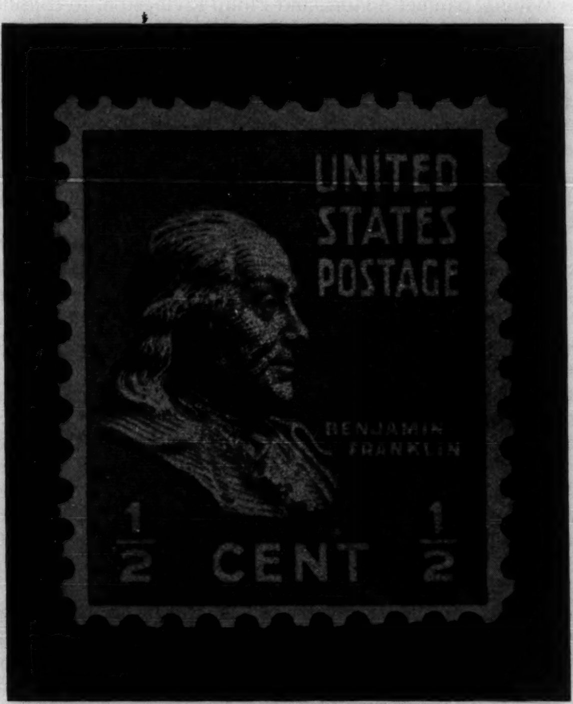
The Duke of Grafton conveyed a warning to him of the danger, suggesting that if Franklin desired to retain his office he should return to America. Grafton was one of those who had formed a high opinion of Franklin's talents, for he was later to suggest that if Franklin should be relieved of his Deputy Postmastership he should be given some other post. Reports of this new post reached America, where Foxcroft spoke of the proposed appointment of Franklin to the office of Under Secretary of State to Lord Hillsborough. For some little time, therefore, Franklin was in the position of not knowing whether he was to be discarded or promoted.

The uncertainty of his future did not prevent Franklin from indulging in a piece of impertinence which may have given him much glee but which certainly would have precipitated his departure from office had it been detected by responsible authorities. As the holder of the franking privilege he had customarily franked his letters with the words "Free. B. Franklin." He now changed the superscription on some letters home to read "B. Free Franklin." Perhaps it was a little too subtle for those who handled the letters.

Then, in 1774, came the affair of the notorious Hutchinson letters. The practice of interfering with letters entrusted to the mail was not such a rare occurrence as the furore raised over this incident would lead one to imagine, but a vast amount of animosity was manufactured against Franklin for his alleged violation of a private correspondence entrusted to the mail, for the prompt and secure conveyance of which he was responsible. What followed is best described in a letter he wrote on February 15, 1774.

> I received a written notice from the secretary of the general post office, that his Majesty's postmaster-general *found it necessary* to dismiss me from my office of deputy postmaster-general in North America. The expression was well chosen, for in truth they were *under a necessity* of doing it; it was not their own inclination; they had no fault to find with my conduct in the office; they knew my merit in it, and that, if it was now an office of value it had become such chiefly through my care and good management, that it was worth nothing when given to me; it would not pay the salary allowed me and, unless it did, I was not to expect it; and that it now produces near three thousand pounds a year clear to the treasury here. They had besides a personal regard for me. But as the post offices in all the principal towns are growing daily more and more valuable, by the increase of correspondence, the officers being paid *commissions* instead of *salaries*, the ministers seem to intend, by directing me to be displaced on this occasion, to hold out to them all as an example that, if they are not corrupted by their office to promote the measures of the administration, though against the interests and the rights of the colonies, they must not expect to be continued. This is the first act to extend the influence of the government in this branch. But as orders have been some time since given to the American postmaster-general, who used to have the disposition of all places under him, not to fill vacancies of value, till notice of such vacancies had been sent hither, and instruction thereupon received from hence, it is plain such influence is to be part of the system; and probable that those vacancies will for the future be filled by officers from this country. How safe the correspondence of your Assembly Committees all along the continent will be through the hands of such officers may now be worth consideration, especially as the post office act of Parliament allows a postmaster to open the letters, if warranted so to do by the order of the Secretary of the State, and every provincial secretary may be deemed a secretary of state in his own province.

Franklin's position in England had been a delicate one. As Deputy Postmaster General he was a Crown official but, as agent for various colonies, he was advancing interests opposed to those of the Crown ministers. If, in his course of actions, he had taken risks, they were calculated risks, and if he felt aggrieved for being dismissed after his long and loyal service to the Post Office, which he had brought from the stage of bankruptcy to one of prosperity, there should have been small reason for surprise.



One-half cent stamp, 1938. From photograph of the James Earle Fraser statue in The Franklin Institute.

In America he was replaced by Hugh Finlay, but Foxcroft was retained in his post. As the latter had now moved to New York, the headquarters of postal affairs was finally transferred to the city of the government's choice. The most significant feature of the change was that the Post Office was regarded as being placed safely under loyalist control. For the colonists objected to using what they called the "royal post" and before long a "constitutional post" was inaugurated by William Goddard of Baltimore. Thus, when Franklin returned to America he found the two postal systems openly competing for business. The newer enterprise soon wiped out the "royal post," and Franklin's name was once more associated with that of the Post Office.

On May 29, 1775, a committee of the Continental Congress was formed, comprising Benjamin Franklin, Thomas Lynch, R. H. Lee, Thomas Willing, Samuel Adams, and Philip Livingston, to consider the matter of the postal service. On July 26th, this committee presented its report, recommending the establishment of a Post Office for the various states. Letter rates were to be 20 per cent lower than those charged by the Act of 1765, but this recommendation was later rejected. Except for the payment of the postmasters, there was very little changed in the system as it had been under Franklin's administration. Franklin was unanimously appointed Postmaster General under the Confederaation at a salary of $1000, but he generously applied the entire amount of his salary to those who were disabled in the war.

The most disappointed man in the series of changes was Goddard. He had anticipated being appointed Comptroller of the service, which was the office next in importance to Franklin's but this post went to the latter's son-in-law Richard Bache. Goddard was rewarded with an inferior post as surveyor of post roads, with a salary of $100. He thereon petitioned Congress for a commission in the army.

Franklin's service with the Post Office was now practically ended, although he retained the position to which he had been appointed. He was shortly to leave for France, entrusting the affairs of the system to Bache. When the United States Post Office was finally organized in 1782, President Washington appointed Samuel Osgood to be the first Postmaster General, but it was the cumulative contributions of Franklin to the service which were adopted and from which the present Post Office has been evolved. He has every right to be regarded as the father of the American Post Office.

Once hostilities opened, the value of Franklin's work became apparent in still another way. The post roads he had developed permitted the rapid recruiting of Washington's army, and later provided for the passage of artillery which would have been immobilized if it had had to depend upon the Indian trails which existed before Franklin's enterprise converted them into roads and highways. This is one of the little known features of the effort that made thirteen insignificant colonies into the most prosperous nation on earth.

Because of his numerous contributions to the postal system, it is most appropriate that the Institute founded to preserve his memory and to perpetuate his work should have in its Museum a representative collection of postage stamps which have been issued by the Post Office of this country. No adhesive stamps were used during Franklin's days. Postage was paid in cash to the postmaster who marked on the letters the amount paid. Most letters were merely sheets of paper, folded and sealed. Sometimes the postmasters used a crude stamp made from wood or cork to specify the postage paid but this was a private innovation and not authorized. The first adhesive stamps, known to collectors as "Postmaster Provisionals" were likewise the result of private initiative, since the postmasters introduced them for their own convenience.

Not until 1847 did the government introduce official adhesive stamps, of 5 and 10 cent values, the former to pay postage up to 300 miles and the latter for longer distances. At first their use was not obligatory. The Post Office recognized Franklin's labors on its behalf by placing his portrait on the popular 5 cent stamp. The stamp of higher value bore the portrait of Washington. It is noteworthy that the portraits of these two founders of the republic are the only two which have appeared upon every definitive issue of American stamps.

Franklin would have been charmed with the idea of the adhesive stamp, apart from its convenience, for these stamps are frequently gems of the printing craftsman's art. The first (1847) stamp bore a portrait of Franklin from a drawing by James B. Longacre, showing him in three-quarters face, wearing a white kerchief and a fur collared coat. In 1851 the government issued a much fuller issue of nine stamps to meet the requirements of a lower postal rate, including the first 1 cent stamp of American use. The 1 cent, 30 cent, and a 1 cent "carrier's" stamp of this series bore the head of Franklin, apparently drawn from Houdon's famous bust. This particular bust continued to be the favored subject for reproduction on all stamps bearing Franklin's likeness until the elaborately designed series of 1902 was issued. The one cent stamp in this series is a miniature reproduction of Duplessis' portrait. An interesting detail on this stamp is provided by two supporting cherubs who hold aloft electric light bulbs. This was probably the first, but by no means the last, philatelic allusion to electricity.

After this the Post Office reverted to the use of the Houdon bust for its Franklin stamps, and the same artist's bust of Washington answered the purpose in the definitive series (there was a special commemorative series which shows twelve different portraits of Washington) whenever the first president had to be portrayed until the current series issued in 1938. This series, commonly called the "Presidentials," is distinguished for the extreme simplicity of design, the large number of stamps in the series, and the subject matter. It embraces portraits of all the presidents from Washington to Coolidge. Three broken-cent values were introduced for convenience, $\frac{1}{2}$, $1\frac{1}{2}$, and $4\frac{1}{2}$ cents, which were allotted to Franklin, Martha Washington, and the White House, respectively. The Franklin stamp which thus stands at the head of the series has an unique interest because of its association with The Franklin Institute.

In the first place, the portrait is a departure from custom since it is taken from the Franklin statue by James Earle Fraser that stands in the Institute building; and secondly, because the stamp was first put on sale at a branch post office set up in the building for that purpose on the day the statue was dedicated and unveiled.

Because of Franklin's intimate association with the postal system and because of his long and influential connection with the graphic arts, it is appropriate that a fine collection of American stamps should be among the Museum's possessions. Such a collection would have been beyond the Institute's financial resources, and its acquisition could only be hoped for through the generosity of a benefactor having the means, the knowledge, and the time to have accumulated a suitable collection. Most collectors of modest means have cause to lament the high values placed upon the rarer stamps and rarely have an opportunity to see a complete collection. The Harry L. Jeffreys collection is not only complete but was assembled after a long and intensive study, so that it includes many examples of uncommon variations which will be of especial interest to advanced philatelic students.

The bequest by Mr. Jeffreys of this valuable collection of American stamps has, therefore, enriched the Institute's material devoted to Franklin and his interests. The collection is now being examined by a group of specialists with a view to placing it on display.

ON THE ASYMPTOTIC EXPANSION IN THREE-DIMENSIONAL COMPRESSIBLE VISCOUS FLOW

BY

M. Z. KRZYWOBLOCKI[1]

SUMMARY

An exact solution of the set of equations (of motion, energy, continuity, and state) of a three-dimensional flow of a compressible viscous fluid (in which pressure, density, coefficients of viscosity and heat conductivity are functions of position) represents insurmountable difficulties and cannot be achieved. Consequently, only approximation methods may be considered. Goldstein (**1, 2**)[2] proposed an asymptotic expansion for the two-dimensional steady flow of an incompressible viscous fluid behind a solid body by means of exponential functions. That type of expansion was applied by the author to a few cases of a two-dimensional steady flow of a compressible viscous flow (**3, 4, 5**). In the present paper the author extends this method to a case of a three-dimensional flow of a compressible fluid. Not to obscure the problem by many items of a simple algebraic nature, the paper presents only a general outline of the method of attack, applied to the simplest possible case, that is, to a flow along a yawed flat plate whose plane is parallel to the direction of the undisturbed flow. From two requirements: (*a*) to solve exactly or approximately the equations, (*b*) to satisfy exactly or approximately the boundary conditions, in a case of a compressible viscous fluid (when no use is made of velocity potential—or stream—functions) it is possible to satisfy the requirement (*a*) approximately and (*b*) exactly. This was shown by the author in the present as well as in previous papers. The presented method may be easily adjusted to three-dimensional wakes and jets similarly as it was done in plane flows.

1. BASIC EQUATIONS AND ASSUMPTIONS

Assuming that the coefficients of viscosity μ and heat conductivity k are variable functions (they may be assumed to be polynomials or power series in T with properly selected coefficients to assure a convergence) one obtains the following equations (of motion, continuity, state, and energy):

$$\rho(u_{i,t} + u_k u_{i,k}) = \tau_{ik,k}, \tag{1}$$

$$\rho_{,t} + (\rho u_i)_{,i} = 0, \qquad p = R\rho T, \tag{2}$$

$$Jc_v\rho(T_{,t} + u_i T_{,i}) + pu_{i,i} = J(kT_{,i})_{,i} + \Phi, \tag{3}$$

$$\Phi = \mu[2u_{j,i}u_{j,i} - (-\textstyle\sum_{\sigma=i,j,k} \varepsilon_{ij\sigma}u_{j,i})^2] - \frac{2}{3}\mu(u_{i,i})^2, \tag{3a}$$

where no extraneous forces are taken into account and the effect of gravitation on the compressibility phenomena is also neglected. The subscripts with a comma denote partial differentiation, the symbol Φ denotes the dissipation function, the skew-symmetric quantities

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

ε_{ijk} equal $+1$ or -1, according as the indices i, j, k are even or odd permutations of the integers 1, 2, 3, otherwise being equal to zero, and the components of the stress tensor are: $\tau_{ii} = -p + \mu(\varepsilon_{ii} - \frac{2}{3}u_{i,i})$, $\tau_{ij} = \mu\varepsilon_{ij}$, $\varepsilon_{ij} = u_{i,j} + u_{j,i}$. A further assumption is that the plate is located very far down-stream behind the origin of the Cartesian coordinates system. In an application of a series expansion in negative powers of x and z this assumption enables one to neglect consistently the higher powers of $(1/x)$ and $(1/z)$. Consequently the expansion is an asymptotic one for large x and z. The axes of the coordinates system are chosen in the following way: the x-axis is horizontal and perpendicular to the leading edge of the yawed plate, the y-axis is vertical and the z-axis is parallel to the leading edge. The horizontal plane (x, z) is the plane of the plate. The origin of the system is assumed to be located far ahead in front of the plate (and below it).

2. STEADY, THREE-DIMENSIONAL FLOW

Expanding the previously cited equations and adding on both sides certain terms ($V_0 d\rho u_{zz}$ in the first, $V_0 d\rho v_{zz}$ in the second, $V_0 d\rho w_{zz}$ in the third equation of motion, $Jdc_v\rho uT_{zz}$ in the equation of energy, where $V_0 = V_0(u_0, v_0, w_0)$ is the uniform velocity of the incoming, undisturbed flow and d is an arbitrarily chosen length) one obtains the following three equations of motion, continuity and energy (with the equation of state included into four of them):

$$\begin{aligned} V_0 d\rho u_{zz} - \mu u_{yy} = {} & - R(\rho T)_x - \rho(uu_x + vu_y + wu_z) \\ & - \tfrac{2}{3}[\mu(u_x + v_y + w_z)]_x + 2(\mu u_x)_x + (\mu v_x)_y + \mu_y u_y \\ & + [\mu(u_z + w_x)]_z + V_0 d\rho u_{zz}, \end{aligned} \tag{4}$$

$$\begin{aligned} V_0 d\rho v_{zz} - \tfrac{4}{3}\mu v_{yy} = {} & - R(\rho T)_y - \rho(uv_x + vv_y + wv_z) + [\mu(u_y + v_x)]_x \\ & - \tfrac{2}{3}[\mu(u_x + w_z)]_y - \tfrac{2}{3}\mu_y v_y + [\mu(v_z + w_y)]_z + V_0 d\rho v_{zz}, \end{aligned} \tag{5}$$

$$\begin{aligned} V_0 d\rho w_{zz} - \mu w_{yy} = {} & - R(\rho T)_z - \rho(uw_x + vw_y + ww_z) \\ & - \tfrac{2}{3}[\mu(u_x + v_y + w_z)]_z + [\mu(u_z + w_x)]_x \\ & + (\mu v_z)_y + \mu_y w_y + 2(\mu w_z)_z + V_0 d\rho w_{zz}, \end{aligned} \tag{6}$$

$$u\rho_x + v\rho_y + w\rho_z = -\rho(u_x + v_y + w_z), \tag{7}$$

$$\begin{aligned} J(dc_v\rho uT_{zz} - kT_{yy}) = {} & - Jc_v\rho(uT_x - duT_{zz} + vT_y + wT_z) \\ & - R\rho T(u_x + v_y + w_z) + J[(kT_x)_x + k_yT_y + (kT_z)_z] + \Phi, \end{aligned} \tag{8}$$

$$\begin{aligned} \Phi = \mu\{ & 2[(u_x)^2 + (v_y)^2 + (w_z)^2] - \tfrac{2}{3}(u_x + v_y + w_z)^2 \\ & + (u_z + w_x)^2 + (u_y + v_x)^2 + (v_z + w_y)^2\}, \end{aligned} \tag{8a}$$

where subscripts denote partial differentiation. A brief explanation of the method of solution of this set of differential equations will be given. The coefficients on the left hand sides of those equations will be assumed so as to obtain ordinary linear differential equations (of the first or second order) with constant coefficients. On the right hand sides the results from the preceding approximations will be used. The results will be presented in forms of series $u = \sum_1^n u_i$, $w = \sum_1^n w_i$, etc. The following substitutions and transformations will be used (based on the well-known Prandtl's transformation used by H. Blasius in 1908):

$$\eta = y(V_0 d)^{1/2}(4\nu_0 xz)^{-1/2}, \qquad \eta_x = -\tfrac{1}{2}\eta x^{-1}, \text{ etc.} \tag{9}$$

3. FIRST APPROXIMATION (TERMS OF ORDER −2 AND −3)

3.1 The x-velocity component

This component will be always calculated from the first equation of motion. In the first approximation substitute on the left side the quantities $\rho = \rho_0$, $\mu = \mu_0$, $u = u_1 \sim x^{-1/2}z^{-1/2}$ (where subscript zero refers to the incoming undisturbed flow) and on the right side put $v = 0$, $w = w_0$, $\mu = \mu_0$, $T = T_0$, $\rho = \rho_0$. Preserve on the right side only the terms of the order $x^{-3/2}z^{-3/2}$ (joint order -3). The boundary conditions are that u tends to zero when $x \to \infty$ or $z \to \infty$ and tends to zero exponentially when $y \to \infty$. For $y = \eta = 0$, u and u_1 must vanish and $u_{1_y} \sim u_{1_\eta}$ should be finite. As the result we obtain:

$$V_0 d\rho_0 u_{1xz} - \mu_0 u_{1yy} = 0. \tag{10}$$

Letting

$$u_1 = x^{-1/2}z^{-1/2}u_0 \exp\,(-\eta)f_1(\eta), \tag{11}$$

and performing all the necessary computations one obtains the equation

$$f_1''(\eta^2 - 1) - f_1'(2\eta^2 - 3\eta - 2) + f_1(\eta - 3)\eta = 0. \tag{12}$$

This may be solved by representing the function f_1 as a power series in η, that is, $f_1 = \sum_0^\infty a_i\eta^i$. The primes denote partial differentiation with respect to η. Boundary conditions require that $a_0 = 0$ and the value of a_1 must be taken from test so as to provide the proper slope of the $u_1(0)_\eta$. We note that in particular cases the component v_0 may vanish.

3.2. The y-velocity component

This will be always calculated from the second equation of motion. To find the first non-vanishing order of v, apply the continuity equation with $u = u_1$, $w = w_0$ and $\rho = \rho_0$. Obtain $u_{1x} + v_y = 0$ which shows that $v \sim x^{-1}$, that is v is of an order lower than u_1. This results in the value $v_1 = 0$. In order to check whether the last value really fulfills the second equation of motion, insert into Eq. 5 the quantities $\rho = \rho_0$, $v = v_1$, $\mu = \mu_0$ on the left side and $\rho = \rho_0$, $T = T_0$, $\mu = \mu_0$, $v = 0$, $w = w_0$, $u = u_1$, on the right side. Preserving only the terms $\sim x^{-3/2}z^{-3/2}$ gives the equation:

$$V_0 d\rho_0 v_{1xz} - \tfrac{4}{3}\mu_0 v_{1yy} = 0. \tag{13}$$

Hence, really, $v_1 = 0$ is a solution.

3.3. *The z-velocity component*

This will be calculated from the third equation of motion. Letting

$$w_1 = x^{-1/2} z^{-1/2} w_0 \exp(-\eta) h_1(\eta), \tag{14}$$

and substituting $\rho = \rho_0$, $u = u_1$, etc. on the right side gives the equation

$$3(V_0 d\rho_0 w_{1xz} - \mu_0 w_{1yy}) = \mu_0 u_{1xz}. \tag{15}$$

The solution of the homogeneous equation is identical with that in Section 3.1, that is, $h_{11} = \sum_1^\infty a_i \eta^i$, $(h_1 = h_{11} + h_{12})$ and with the boundary conditions similar to those for the u-component. A particular solution h_{12} may be obtained from the equation

$$\begin{aligned} &[h_{12}''(\eta^2 - 1) - h_{12}'(2\eta^2 - 3\eta - 2) + h_{12}(\eta^2 - 3\eta)] \\ &\qquad = A_1[f_1''\eta^2 - f_1'(2\eta - 3)\eta + f_1\eta^2], \\ &\qquad\qquad A_1 = \mu_0 u_0 (3V_0 d\rho_0 w_0)^{-1}. \end{aligned} \tag{16}$$

The function h_{12} may be represented by a power series $h_{12} = \sum_0^\infty c_i \eta^i$. Boundary conditions require that $c_0 = 0$ and $(a_1 + A_1 c_1)$ must be chosen so as to assure the proper slope $w_1(0)_\eta$. It may happen that both curves $u_1(\eta)$ and $w_1(\eta)$ have the same slope at $\eta = 0$, which would require $c_1 = 0$. But this is a particular item without any influence on the procedure of calculation.

3.4. *Density*

This will be calculated from the continuity equation in which on the right side the terms of a joint order -2 are preserved. The boundary conditions are: $\rho = \rho_0$ for $x \to \infty$, $y \to \infty$ and $z \to \infty$ and $\rho = \rho_s$ (surface) for $\eta = 0$. The condition $\rho(0) = \rho_s$ must be preserved whether the fluid is in motion or at rest (the plate may be heated). Hence put $\rho = \rho_0 + (\rho_s - \rho_0)\exp(-\eta) + \rho_1$. On the left side of the continuity equation put $u = u_0$, $v = 0$, $w = w_0$, $\rho = \rho_1$, and on the right side $\rho = \rho_0$, $u = u_1$, $w = w_1$. Obtain the equation:

$$u_0 \rho_{1x} + w_0 \rho_{1z} = -\rho_0(u_{1x} + w_{1z}). \tag{17}$$

Preserving the terms $\sim x^{-3/2} z^{-1/2}$ gives the result $\rho_{11} = -\rho_0 u_0^{-1} u_1$. Similarly $\rho_{12} = -\rho_0 w_0^{-1} w_1$ (terms $\sim x^{-1/2} z^{-3/2}$) and the final result is $\rho_1 = \rho_{11} + \rho_{12}$. The constants of integration equal zero to fulfill the boundary condition that $\rho_1 = 0$ for $\eta \to \infty$. Perhaps a remark should be made that, in order to obtain Eq. 17 with the use of the final value for ρ_1, during a differentiation in the x-direction only the part of ρ_1 containing u_0 is subject to variation etc. There cannot be in the value for ρ_1 the terms $\sim x^{1/2}$ or $\sim z^{1/2}$ as they do not satisfy the boundary conditions.

3.5. *Temperature*

This will be calculated from the energy equation. The boundary conditions for T are similar to those for ρ. Hence put $T = T_0 + (T_s - T_0) \exp(-\eta) + T_1$. On the right side of Eq. 8 put the quantities $\rho = \rho_1$, $u = u_1$, $v = 0$, $w = w_1$, $T = T_0$, $\mu = \mu_0$, $k = k_0$ and on the left side the quantities $\rho = \rho_0$, $u = u_0$, $T = T_1 = T_{11} + T_{12}$. Preserving the terms $\sim x^{-2} z^{-1}$ gives the value of $T_{11} = x^{-1} z^0 \exp(-\eta) t_{11}(\eta)$. Similarly the quantity $T_{12} = x^0 z^{-1} \exp(-\eta) t_{12}(\eta)$ may be obtained (terms $\sim x^{-1} z^{-2}$). The part T_{11} satisfies the boundary conditions although it is proportional to z^0 since for $z \to \infty$, η approaches to zero and the power expansion for the function t_{11} does not possess a term containing η^0. Consequently $T_{11 z \to \infty} = 0$.

4. SECOND AND HIGHER APPROXIMATIONS

The second and higher approximations may be found in a similar way. For illustrative purposes let us discuss briefly the second approximation of u. Put on the left side the quantities $\rho = \rho_0$, $\mu = \mu_0$, $u = u_2$ and on the right side the quantities $\rho = \rho_0 + \rho_1$, $u = u_1$, $v = 0$, $w = w_1$, $T = T_0 + T_1$. Preserve the terms of a joint order -4 but of a single order lower than or equal to -1 (that is, $\sim x^{-3} z^{-1}$, $\sim x^{-5/2} z^{-3/2}$, $x^{-2} z^{-2}$, $x^{-3/2} z^{-5/2}$, $x^{-1} z^{-3}$). The solution may be obtained in a way similar to that for T_1 and u_2 will be composed of several parts ($u_{21} \sim x^{-2} z^0$, $u_{22} \sim x^{-3/2} z^{-1/2}$, $u_{23} \sim x^{-1} z^{-1}$, etc.).

REFERENCES

(1) S. GOLDSTEIN, "On the Two-Dimensional Steady Flow of a Viscous Fluid behind a Solid Body," *Proc. Roy. Soc. of London*, A, Vol. 142, pp. 545–562 (1933).

(2) S. GOLDSTEIN, "Modern Developments in Fluid Dynamics," Oxford, Clarendon Press, 1938.

(3) M. Z. KRZYWOBLOCKI, "On The Two-Dimensional Steady Flow of a Compressible Viscous Fluid Far Behind a Solid Symmetrical Body," JOURNAL FRANKLIN INSTITUTE, Vol. 243, No. 6, pp. 471–486 (1947).

(4) M. Z. KRZYWOBLOCKI, "On Steady, Laminar Two-Dimensional Jets in Compressible Viscous Gases Far Behind the Slit." *Quar. Appl. Math.*, Vol. 7, pp. 313–323 (1949).

(5) M. Z. KRZYWOBLOCKI, "On Steady, Laminar, Round Jets in Compressible Viscous Gases far Behind the Mouth." *Oesterreich. Ing. Archiv.*, Vol. 3, pp. 373–383 (1949).

Telephoned Television: Communication of Tomorrow.—Recently, a new *wired* television system called Vericon was announced by Remington Rand Inc. It differs from commercial television in that its images are transmitted by coaxial cable, instead of broadcast through the air. The system is being used right now in fields like medical training, jet motor manufacture, and by the Army for the defusing of live bombs.

The following indicates a few applications now in operation. The Medical School of the University of Kansas has installed Vericon as a regular surgical training technique which permits dozens of students sitting in an auditorium hundreds of feet away from the operating room to observe delicate operations. Army Ordnance Engineers have found this wired television system invaluable in the work of defusing live bombs and projectiles. For several years they have been using a remote control wrench for this delicate operation, while a Vericon camera sends a picture of what is happening over a cable into a bomb-proof shelter. By watching the television screen they can operate the remote control mechanism—thus eliminating a hazardous task that could mean death even for experts.

A spectacular example of wired television in use recently took place in a west coast jet and rocket manufacturer's plant. This company had developed a new jet motor at a cost that ran into six figures. In preparing for its first ground test, engineers calculated that there was a definite explosion hazard. To protect themselves, and to still see what was happening, they devised a concrete wall and a system of mirrors that would permit them to watch the motor from the other side of the barrier. They also installed Vericon, in addition to the mirrors, on an experimental basis. Thirty seconds before the initial blast of the motor, all employees were ordered to stand behind the wall. Almost at the instant that the jet was to fire, the man at the television screen noticed a fuel leak in one of the valves. The detail was too small to be detected in the mirrored image. The test was instantly halted and repairs were made—saving almost certain destruction of the expensive jet prototype.

What can we expect tomorrow from wired television? One scientist, William Norvell, former Army Air Force television expert, now with Remington Rand, foresees the probability that a housewife will be able to phone her favorite department store and examine merchandise as it appears—in color—on her telephone's viewing screen. He also conceives of a student in California who will be able to study archives of the Smithsonian Institution in Washington, D. C. simply by dialing "long distance." One of the world's greatest department stores is thinking of installing Vericon cameras around its floors to watch for shoplifters. Another researcher suggests the possibility of an arrangemnet with your local movie house whereby you'll be able to dial them and watch the evening show in the comfort of your own living room.

Other uses? Mr. Norvell reports that Remington Rand has received hundreds of suggestions for new uses for wired television. He says they include ideas like monitoring in tunnels and on bridges, and for signature identification in banks or retail stores.

ON CERTAIN MATTERS PERTAINING TO ELECTRETS

BY

W. F. G. SWANN[1]

ABSTRACT

The object of the paper is to trace the mathematical consequences of the assumption that an electret consists of (1) a distribution of polarization which decays with time, (2) a distribution of surface and volume charge which disappears according to ohmic conductivity having no relation to the decay of the polarization.

The electret is supposed provided with metal plates permanently attached to its surface and to have a thickness of 1 cm. and a radius of 3 cm. Starting with one plate at zero potential and the other at a negative potential V_0, symbolizing in the ordinary terminology the condition in which the electret shows a "heterocharge," the absolute value of V decays with time to zero. It then rises to a positive maximum, after which it again decays with time, ultimately to zero. It turns out that the functional relation between potential difference and time is absolutely independent of the charge distribution within the electret and of its decay. The only parameters involved are the resistance R connecting the plates and the parameter determining the rate of decay of the polarization. However, while the initially produced polarization plays a part in determining the maximum positive potential V_m attained and while V_0 plays a part in determining the time for the potential to change from V_0 to zero, neither of these quantities plays any part in determining the subsequent time for rise to the maximum V_m. If the resistance is such as would be determined by ordinary ohmic conductivity in a substance of the normal specific resistance of a wax, if the polarization decays according to a factor $e^{-\alpha t}$, and if α is 10^{-8} sec.$^{-1}$, corresponding to a decay of the polarization to $1/e$ of its initial value in about three years, then, starting with $V_0 = -2000$ volts and with the initially produced polarization such as to make $V_m = 2000$ volts, there results the following: V will change from its original value -2000 volts to zero in about one and a half days, then it will rise to the positive maximum V_m in about two weeks, after which it will decay to one third of that maximum value in about three years.

If the electret be exposed to the open air, the conductivity of the air would produce a smaller resistance than the wax itself, and the maximum would be attained in about 42,000 seconds after the value $V = 0$ was reached.

If, with the conductivity of the wax alone operating in determining R, we desire a maximum value of V equal to 2100 volts, it would only be necessary to have an initial polarization less than 1 per cent of that which would be produced by a complete orientation of the dipole moments of the molecules of the wax in one direction.

Expressions are also developed for the case where the charge within the electret disappears by self-repulsion to the plates, in addition to its decay by ohmic conductivity. Such a phenomenon can, of itself, simulate a decaying polarization.

A discussion of experiments pertaining to the field outside of the electret is also given.

INTRODUCTION

Statements regarding the behavior and the theory of the behavior of electrets have wide variations and are frequently vague in the precise meanings to be attached to such concepts as charge, polarization, etc. However, ideas seem to have crystalized themselves into explanations of the phenomena in terms of two separate mechanisms:

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1. A polarization phenomenon, or its equivalent, which gives rise to what, with some confusion of terminology, is spoken of as a "heterocharge," and which decays with time.
2. A "homocharge" which appears to be regarded as a real charge of electricity and which, somehow or other, gets into the body of the dielectric during the formation of the electret. It is usually regarded as a layer of positive electricity[2] existing near to the surface adjacent to the positive plate of the condenser system used in forming the electret. Presumably there is sense to supposing the possible existence (depending upon the circumstances) of a negative "homocharge" adjacent to the surface which itself is adjacent to the negative plate of the condenser during the formation of the electret.

It is generally supposed that the "homocharge" does not decay with time or that, if it does, it decays more slowly than the polarization associated with the "heterocharge" phenomenon.

In the discussion of these matters, there frequently appears to be considerable vagueness as to certain consequences which must inevitably follow from a straightforward application of the known laws of electrostatics and of electrical conductivity inherent in the atmosphere, as well as in the dielectric itself. Thus, suppose that, as a result of charges or polarization in the dielectric, we have an external field—and it is, of course, only through the existence of such a field that the said charges or polarization can be detected. Then if, for example, the dielectric were exposed to the free air, a large fraction of the field would disappear in a short time as a result of atmospheric conductivity[3] if there were no means of continually recreating the field. In virtue of its conductivity,

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[2] Such is, indeed, the view of B. Gross. For a general survey of the subject, including an extensive bibliography, the reader is referred to a paper "The Electret" by F. GUTMANN, *Rev. Mod. Phys.*, Vol. 20, p. 457 (1948).

[3] In a paper entitled "The Theory of Electrical Dispersion into the Free Atmosphere, with a Discussion of the Theory of the Gerdien Conductivity Apparatus, and of the Theory of the Collection of Radioactive Deposit by a Charged Conductor" (*Terr. Mag.*, Vol. 19, p. 81 (1914)) the present writer has shown that in the case of a body of any shape exposed to air motion of any kind in a space which is not confined, the body with charge Q will lose charge at a rate given by

$$-\frac{dQ}{dt} = 4\pi Qnev,$$

where n is the number of ions per cc. of sign opposite to that of the charge on the body, e is the electronic charge, and v is the mobility of the ions. The formula is a generalization of a similar formula deduced by Riecke (*Annalen der Physik*, Vol. 12, p. 68 (1903)) for the case of a sphere in a uniformly moving stream of air. It applies, under suitable conditions, not only to the charged body as a whole, but to any portion of it. Thus, in the case of a dielectric charged with different signs of charges in different places, it operates to annul the field everywhere.

It may be stated that for normal air, $n \sim 1000$, $v = 450$ cm./sec. per e.s.u. per cm., and $e = 4.8 \times 10^{-10}$ e.s.u., so that $4\pi nev \sim 2.6 \times 10^{-3}$.

the air acts, in fact, as a leakage resistance; and in the appendix to this paper, Problem 2, the matter is treated in detail from this aspect. It is true that under laboratory conditions an electret is not exposed to the "free air" in the above sense. All sorts of "saturating fields" are apt to be about, which fields tend to rob the air of its ions. However, without specification of the conditions, it is impossible to say how much atmospheric conductivity plays a part in the sense indicated. It is to be observed, moreover, that, *as regards its external effects*, by which it is measured, a charge embedded inside the dielectric is not protected from the effect of atmospheric leakage, even though the dielectric may have infinite resistivity. As long as the internal charge gives rise to an external field, that field will bring ions to the surface in such a manner as finally to reduce all external fields to zero unless there is operative some agency which recreates the field.

In the conventional view as to the operation of an electret, we have a large polarization which decays with the time and this fact, in combination with the other attendant circumstances to be more specifically outlined below, gives rise to a condition in which an external field can exist in semi-permanent form.

It is convenient to think of a situation in which the electret has fastened to each side a piece of metal foil. Suppose we connect the two foils together. Then, in this condition, we have zero field external to the electret. On the conventional view, which regards the homocharge as a layer of electricity inside the dielectric, we can then look upon the "zero field" cited above as made up out of the following parts:

(*a*) The field due to the polarization.

(*b*) The field due to the "real" homocharge.

(*c*) The field due to any additional charges which have come to the plates in order to secure initial absence of potential difference between them. We may call this contribution the "pseudo homocharge," since it is indistinguishable from the real homocharge as regards its external electrical effects.

If the short-circuiting wire is removed from the foils, then, as the polarization decreases with time, the field due to (*b*) and (*c*) makes itself evident. There is no way of separating the effects (*b*) and (*c*) if we confine ourselves to the observation of external phenomena. Indeed, if there were no real homocharge, the "pseudo homocharge" (*c*) would give rise to all those phenomena involving external fields which are characteristic of the behavior of the electret.[4]

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[4] Even if there are no foils attached to the electret, the charge of the type (*c*) will make its appearance from ions in the air, or from natural leakage in the dielectric itself; and if the polarization and real homocharge were constant with time, the charge of type (*c*) would finally adjust itself to the same value as that corresponding to the case where there were foils which were connected together. In practice, the charge (*c*) will build up to an extent determined by the rate of change of polarization and the leakage, through air or other medium, in a manner which will be developed in more exact detail in what follows. (Cont. next page.)

It may be remarked that Dr. H. O. Albrecht, following the writer's suggestion, has made an electret with foils permanently attached to the two sides of the wax before the field used to make the electret was applied[5] and he has treated this electret for measurements exactly as though it were an electret prepared in the ordinary manner. In other words, he has, with the usual precautions, measured the inductive effect produced when a brass plate *C*, Fig. 1, was brought near to the top surface, then connected to *B*, then insulated, and finally removed to a distance with observation of the resulting electroscope deflection.

Starting with the top foil insulated immediately after having been connected to the lower foil, the operation with the plate *C* in the above experiment gave no deflection at first. On repetition of the experiment after a suitable lapse of time, with F_1 insulated, an effect was observed of such sign as to correspond to the usual sign attributed to the growth of homocharge in the electret, and this effect grew with lapse of time. Moreover, the electret showed an apparent homocharge in the above sense whichever side of the electret was uppermost in the experiment, one apparent homocharge being positive and the other negative. In fact, such electrets, with foils permanently attached, behaved in a more consistent manner than electrets without foils, as indeed they should.[6]

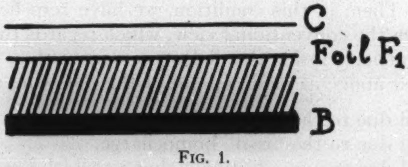

FIG. 1.

In what follows, we shall concern ourselves with the case where any real charge density distribution within the dielectric disappears with time purely as a result of an Ohm's law conductivity, while the semi-permanent polarization decays with time according to independent causes, such, for example, as may be symbolized by an exponential decay $P_x = P_{0x}e^{-\alpha t}$, where α has no explicit relation to the conductivity. In the particular case where the internal ohmic conductivity is zero, the internal charge density will, of course, remain constant with time. Later we shall discuss briefly the case where the decay of the real charge

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It may be remarked that passing a flame over an electret brings it almost instantaneously to the condition corresponding to the case where it has connected foils. From this condition, if left to itself, it proceeds to develop activity with time on account of the decay of the polarization.

[5] The electret was, of course, made in the solid state.

[6] It must be confessed that it is difficult to see how one could expect results other than those cited. However, electret phenomena have been surrounded with such mystery that there is not a consensus of opinion as to what is obvious and the writer has found some, to whom he has talked, surprised by the results described above.

density is governed, not only by ohmic conductivity, but also by the motion of the charge to the plates by virtue of self-repulsion.

It will appear that when the disappearance of the charge in or on the surface of the electret is governed only by a pure ohmic conductivity, the story of the rise and fall of potential difference between the two faces of the electret is explicitly independent of anything to do with the charge distribution and will depend only upon the initial difference of potential, the decay of the polarization and an effective leakage resistance between the two faces. In the case where Ohm's law is not obeyed and where there is motion due to self-repulsion, such motion can simulate a decaying polarization.

We shall now work out in mathematical detail problems designed to illustrate the foregoing remarks.

MATHEMATICAL

The problem to be attacked may be specified as follows: We have a slab of dielectric material between two parallel plates A and B, designated alternately by subscripts (1) and (2). These plates are kept joined by an external resistance R_e, which may be infinite; and, as will be seen, the internal conductivity determines an internal resistance. The material has a semi-permanent polarization P_x which may vary with position between the plates so long as it exists in planes parallel to the plates. However, the polarization shall decay with time according to some law which, for mathematical convenience, may be taken as $P_x = P_{0x}e^{-\alpha t}$, although, of course, the general discussion is not confined to this special case. The positive direction of the axis of x shall be from A to B. There shall be additional polarization induced by the field in the amount $(K - 1)E/4\pi$, where K is the specific inductive capacity, here defined as the ratio of the change of the dielectric displacement D to the change of field E which causes it.

We shall suppose that, in addition, the dielectric contains a semi-permanent charge distribution which depends only upon x and which is constant with the time or decays according to a pure ohmic conductivity or according to some other law. In the simple case envisaged by Gross[2] this charge would be supposed to exist approximately in a plane near to one of the surfaces of the dielectric. For generality we shall suppose that it is distributed in an arbitrary manner throughout the dielectric, subject only to the restriction that the density is a function only of x.

The end result of the present problem is to find the expression for the potential V of the plate A as a function of the time,[7] starting with the condition that at $t = 0$, $V = V_0$. We shall also seek other information of general interest in the problem.

We shall be concerned with[8] the resultant polarization

$$P' = P + (K - 1)E/4\pi, \qquad (1)$$

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[7] The plate B shall be taken arbitrarily as at zero potential.

[8] We shall omit the subscript x belonging to P when the significance is obvious.

the real volume-charge density ρ, the real surface densities σ_1 and σ_2 at the plates, the fictitious volume-charge density

$$\rho_f = -\operatorname{div} P' = -\partial P'/\partial x, \tag{2}$$

the fictitious surface densities

$$\sigma_{f1} = P'_{n1}; \qquad \sigma_{f2} = P'_{n2}, \tag{3}$$

where the P'_{n1} and P'_{n2} refer to the values in the outwardly directed normal from the medium.

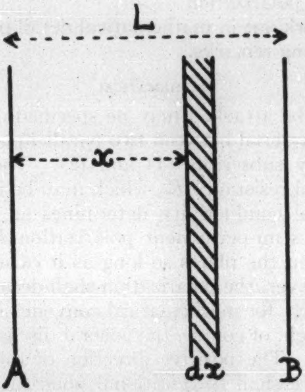

FIG. 2.

We have, of course, quite generally

$$4\pi\rho = \operatorname{div} D = \frac{\partial D}{\partial x}. \tag{4}$$

Now

$$V = V_1 - V_2 = W_V + W_S, \tag{5}$$

where V_2 may be taken arbitrarily as zero, and where

W_V = contribution to $V_1 - V_2$ by the real and fictitious volume densities,

W_S = contribution to $V_1 - V_2$ by the real and fictitious surface densities.

Now, considering a strip of thickness dx, the contribution of the strip to W_V is

$$\begin{aligned} dW_V &= -2\pi x(\rho + \rho_f)dx + 2\pi(L - x)(\rho + \rho_f)dx \\ &= -2\pi(\rho + \rho_f)(2x - L)dx, \end{aligned}$$

where $(\rho + \rho_f)$ refers, of course, to the strip.

Thus

$$W_V = -2\pi \int_0^L (\rho - \operatorname{div} P')(2x - L)dx, \tag{6}$$

where we have recognized (2).

For W_S, we have

$$\begin{aligned} W_S &= 2\pi(\sigma_1 - \sigma_2)L + 2\pi(\sigma_{f1} - \sigma_{f2})L \\ W_S &= 2\pi(\sigma_1 - P'_1 - \sigma_2 - P'_2)L, \end{aligned} \tag{7}$$

where P'_1 and P'_2 are here measured in the positive direction of the axis of x. Now, quite generally

$$D = E + 4\pi P' \tag{8}$$

so that (1) yields

$$\begin{aligned} P' &= P + \left(\frac{K-1}{4\pi}\right)(D - 4\pi P') \\ P' &= \frac{P}{K} + \frac{(K-1)}{4\pi K} D. \end{aligned} \tag{9}$$

Thus, since $\operatorname{div} D = 4\pi\rho$

$$\operatorname{div} P' = \frac{\operatorname{div} P}{K} + \left(\frac{K-1}{K}\right)\rho.$$

Hence, from (6)

$$W_V = -\frac{2\pi}{K}\int_0^L (\rho - \operatorname{div} P)(2x - L)dx. \tag{10}$$

Also, if the field outside of the electret condenser is small compared with the field inside,

$$4\pi\sigma_1 = D_1; \qquad 4\pi\sigma_2 = -D_2 \tag{11}$$

so that, from (9)

$$P'_1 = \frac{P_1}{K} + \left(\frac{K-1}{K}\right)\sigma_1; \qquad P'_2 = \frac{P_2}{K} - \left(\frac{K-1}{K}\right)\sigma_2.$$

Hence (7) yields

$$\begin{aligned} W_S &= 2\pi\left[\sigma_1 - \sigma_2 - \frac{1}{K}(P_1 + P_2) - \left(\frac{K-1}{K}\right)(\sigma_1 - \sigma_2)\right]L \\ W_S &= \frac{2\pi}{K}[\sigma_1 - \sigma_2 - (P_1 + P_2)]L \end{aligned} \tag{12}$$

and from (5), (10) and (12)

$$\begin{aligned}V_1 - V_2 = V &= -\frac{2\pi}{K}\int_0^L\left(\rho - \frac{\partial P}{\partial x}\right)(2x - L)dx \\ &\qquad + \frac{2\pi L}{K}(\sigma_1 - \sigma_2) - \frac{2\pi L}{K}(P_1 + P_2) \\ &= -\frac{4\pi}{K}\int_0^L x\rho dx + \frac{2\pi L}{K}\int_0^L \rho dx \\ &\quad + \frac{4\pi}{K}P_2L - \frac{4\pi}{K}\int_0^L Pdx - \frac{2\pi L}{K}(P_2 - P_1) \\ &\qquad + \frac{2\pi L}{K}(\sigma_1 - \sigma_2) - \frac{2\pi L}{K}(P_1 + P_2)\end{aligned}$$

$$V_1 - V_2 = V = \frac{2\pi L}{K}\left[\sigma_1 - \sigma_2 - \frac{2}{L}\int_0^L Pdx + \int_0^L \rho dx - \frac{2}{L}\int_0^L x\rho dx\right]. \qquad (13)$$

Case of Ohmic Conductivity

The development so far is independent of the law according to which the charge disappears. We shall first consider the case where disappearance of charge is determined by a pure ohmic conductivity.

Now if λ is the specific conductivity, we have the following equivalent expressions

$$i = \frac{d\sigma_1}{dt} + \lambda E_1; \qquad i = -\frac{d\sigma_2}{dt} + \lambda E_2 \qquad (14)$$

for the current density i flowing into the plate A from the external resistance R_e or out of the plate B into the external resistance. Thus

$$2i = \frac{d}{dt}(\sigma_1 - \sigma_2) + \lambda(E_1 + E_2). \qquad (15)$$

Now, from (8), supplemented by (1)

$$D = E + 4\pi P + (K - 1)E = KE + 4\pi P \qquad (16)$$

so that, from (15) and (16)

$$2i = \frac{d}{dt}(\sigma_1 - \sigma_2) + \frac{\lambda}{K}[D_1 + D_2 - 4\pi(P_1 + P_2)].$$

Using (11)

$$2i = \frac{d}{dt}(\sigma_1 - \sigma_2) + \frac{4\pi\lambda}{K}(\sigma_1 - \sigma_2) - \frac{4\pi\lambda}{K}(P_1 + P_2).$$

Now, if I is the current in R_e from A to B, and if A is the area of the plates

$$-\frac{2V}{AR_e} = \frac{2I}{A} = 2i = \frac{d}{dt}(\sigma_1 - \sigma_2) + \frac{4\pi\lambda}{K}(\sigma_1 - \sigma_2) - \frac{4\pi\lambda}{K}(P_1 + P_2).$$

Inserting the value of $(\sigma_1 - \sigma_2)$ from (13)

$$-\frac{2V}{AR_e} = \frac{K}{2\pi L}\frac{dV}{dt} + \frac{2}{L}\int_0^L \frac{\partial P}{\partial t}\,dx - \int_0^L \frac{\partial \rho}{\partial t}\,dx + \frac{2}{L}\int_0^L x\frac{\partial \rho}{\partial t}\,dx$$
$$+ \frac{2\lambda}{L}V + \frac{8\pi\lambda}{KL}\int_0^L P dx - \frac{4\pi\lambda}{K}\int_0^L \rho dx$$
$$+ \frac{8\pi\lambda}{KL}\int_0^L x\rho dx - \frac{4\pi\lambda}{K}(P_1 + P_2). \quad (17)$$

Since the capacity C of the condenser is given by

$$C = \frac{AK}{4\pi L}, \quad (18)$$

$$\frac{dV}{dt} + \left(\frac{1}{R_e C} + \frac{4\pi\lambda}{K}\right)V + \frac{4\pi}{K}\int_0^L \frac{\partial P}{\partial t}\,dx - \frac{2\pi L}{K}\int_0^L \frac{\partial \rho}{\partial t}\,dx$$
$$+ \frac{4\pi}{K}\int^L x\frac{\partial \rho}{\partial t}\,dx + \frac{16\pi^2\lambda}{K^2}\int_0^L P dx - \frac{8\pi^2\lambda L}{K^2}(P_1 + P_2)$$
$$- \frac{8\pi^2\lambda L}{K^2}\int_0^L \rho dx + \frac{16\pi^2\lambda}{K^2}\int_0^L x\rho dx = 0. \quad (19)$$

Now, from the equation of continuity, if i_i is the *internal* current density

$$\frac{\partial \rho}{\partial t} + \operatorname{div} i_i = 0 \quad (20)$$

$$\frac{\partial \rho}{\partial t} + \lambda \operatorname{div} E = 0.$$

Using (16)

$$\frac{\partial \rho}{\partial t} + \frac{\lambda}{K}[\operatorname{div} D - 4\pi \operatorname{div} P] = 0.$$

Since $\operatorname{div} D = 4\pi\rho$,

$$\frac{\partial \rho}{\partial t} + \frac{4\pi\lambda}{K}\rho - \frac{4\pi\lambda}{K}\operatorname{div} P = 0. \quad (21)$$

We can use this expression to simplify some of the contributions to (19). Thus

$$\int_0^L \left(x\frac{\partial \rho}{\partial t} + \frac{4\pi\lambda x}{K}\rho\right)dx = \frac{4\pi\lambda}{K}\int_0^L x\frac{\partial P}{\partial x}\,dx$$
$$= \frac{4\pi\lambda P_2 L}{K} - \frac{4\pi\lambda}{K}\int_0^L P dx; \quad (22)$$

also,

$$\int_0^L \left(\frac{\partial \rho}{\partial t} + \frac{4\pi\lambda}{K}\rho\right)dx = \frac{4\pi\lambda}{K}\int_0^L \frac{\partial P}{\partial x}\,dx = \frac{4\pi\lambda}{K}(P_2 - P_1). \quad (23)$$

Thus (19) yields

$$\frac{dV}{dt} + \left(\frac{1}{R_eC} + \frac{4\pi\lambda}{K}\right)V + \frac{4\pi}{K}\int_0^L \frac{\partial P}{\partial t}\,dx = 0. \tag{24}$$

Now

$$\frac{4\pi\lambda}{K} = \left(\frac{4\pi L}{AK}\right)\left(\frac{A\lambda}{L}\right) = \frac{1}{R_dC},$$

where R_d is the ordinary ohmic resistance of the dielectric considered as a simple material without regard to any charge distributions which it may contain within its substance or on its surface. Thus, since

$$\frac{1}{R} = \frac{1}{R_d} + \frac{1}{R_e},$$

where R is the total resistance of R_d and R_e in parallel, (24) becomes

$$\frac{dV}{dt} + \frac{V}{RC} + \frac{4\pi}{K}\int_0^L \frac{\partial P}{\partial t}\,dx = 0. \tag{25}$$

The most curious feature of this expression is the fact that it is absolutely independent of the initial distribution of charge, and is in fact the same as if the charges in or on the dielectric were zero everywhere. This surprising conclusion is merely a generalization of a case studied by the writer many years ago,[9] in which it was shown that an insulated charged plate embedded in a dielectric between two plates A and B, one (B) of which was connected to the quadrant of an electrometer while the other (A) was connected to the case of the electrometer, resulted in no change of potential of B with time for a case where the conduction of electricity took place in accordance with Ohm's law.

The solution of (22) is

$$V = -\frac{4\pi}{K}\left[e^{-t/RC}\int_0^L dx \int e^{t/RC}\frac{\partial P}{\partial t}\,dt + Ne^{-t/RC}\right], \tag{26}$$

where N must be chosen so that $V = V_0$ when $t = 0$.

Equation 25 and the solution (26) are applicable for any variation of P with time. It is now convenient to assume

$$P = P_{ox}e^{-\alpha t} \tag{27}$$

and (26) yields

$$V = \frac{4\pi\alpha}{K(1/RC - \alpha)}\left[e^{-\alpha t} - e^{-t/RC}\right]\int_0^L P_{ox}dx + V_0e^{-t/RC},$$

where N has been determined to satisfy $V = V_0$ when $t = 0$

$$V = \frac{4\pi\alpha RC}{(1 - RC\alpha)K}\left[e^{-\alpha t} - e^{-t/RC}\right]\int_0^L P_{ox}dx + V_0e^{-t/RC}. \tag{28}$$

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[9] W. F. G. Swann, *Phil. Mag.*, S. 6, Vol. 26, p. 678 (1913).

We shall see that if $\int_0^L P_{ox}dx$ is positive, and if V_0 is negative,[10] then V diminishes in absolute value, passes through zero, then mounts to a positive maximum V_m, and eventually decays to zero. From an observational standpoint, therefore, and in terms of the customary terminology, a negative heterocharge will first be observed, and the decay of this will be followed by the apparent appearance of a positive homocharge. This charge was, in part, present after the removal of the charging potential, when it was masked by the negative polarization. However, an addition really comes through the external or internal resistance from the other plate or from the surroundings through the air during the period when V is negative. All of this is, however, independent of the nature of the real charge distribution in the body of the electret or on its surface, and it is also independent of the time changes of these charges so long as these changes follow Ohm's law at each point. All is governed by the rate of decay of the polarization, which depends upon circumstances having no fundamental relation to the decay of the real charge.

It is of interest to determine the time τ_1 for the potential V to change from the negative value V_0 to zero, and the subsequent time τ_2 for it to rise to the positive maximum V_m. Matters will develop in the most convenient mathematical order by determining τ_2 first.

Determination of the time τ_2 for V to change from zero to its positive maximum V_m, and the determination of that maximum

The desired value τ_2 is obtained most easily by putting $V_0 = 0$ in (28), differentiating with regard to t and equating to zero, after substituting τ_2 for t. In this way we obtain

$$-\alpha e^{-\alpha\tau_2} + \left(\frac{1}{RC}\right) e^{-\tau_2/RC} = 0$$

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[10] If, after the formation of the electret, the positive plate is removed, V_0 depends, in the present view, upon the amount of surface charge which becomes removed by the plate. If it is all removed, there remains only the fictitious surface charge representative of the polarization and this fictitious surface charge will be negative. Insofar as the leakage is determined by the air conductivity as distinct from any ordinary resistance, the potential would cease to remain constant over the surface, since the field over the surface of an electret of finite area would vary with the distance from the axis of symmetry, and the charge per unit area brought to the surface per second by the field would therefore vary with the aforesaid distance from the axis in the case where the electret was exposed to the free air. The leakage contributed by the free air would be quite considerable, as we shall presently see—much greater than that due to ordinary internal leakage. In practice, however, if the electret is in a confined space, as it usually is, or if it is in a vacuum, the air leakage becomes negligible. However, while, for these reasons, the question of air leakage is usually purely academic, we may avoid its complicating our problem unnecessarily by supposing that immediately after the removal of the positive plate used in the formation of the electret we cover the surface with a foil, initially uncharged, using insulated tools for sticking it on. The constancy of potential over the surface will then be guaranteed.

so that

$$\alpha RC = e^{-(1/RC-\alpha)\tau_2}. \tag{29}$$

If we substitute in (28) with $V_0 = 0$, we find, for the maximum V_m, the value

$$V_m = \frac{4\pi RC\alpha}{K(1 - RC\alpha)} \left[(RC\alpha)^{\frac{RC\alpha}{1-RC\alpha}} - (RC\alpha)^{\frac{1}{1-RC\alpha}}\right] \int_0^L P_{ox}dx.$$

After a little reduction, this yields

$$V_m = \frac{4\pi}{K} (RC\alpha)^{\frac{1}{1-RC\alpha}} \int_0^L P_{ox}dx. \tag{30}$$

It is natural to infer, and is indeed true, that if we vary R, the greatest value of V_m is attained when R is greatest.

Determination of the time τ_1 for V to change from the original negative value V_0 to zero

The value τ_1 is obtained by substituting for t in the right-hand side of (28) and equating to zero. The result is given by

$$\frac{4\pi RC\alpha}{(1 - RC\alpha)K}\left[e^{-\alpha\tau_1} \int_0^L P_{ox}dx\right] + \left[V_0 - \frac{4\pi RC\alpha}{(1 - RC\alpha)K} \int_0^L P_{ox}dx\right] e^{-\tau_1/RC} = 0.$$

After a little reduction, this yields

$$\frac{\tau_1}{RC}(1 - RC\alpha) = \log_e \left[- \frac{V_0K(1 - RC\alpha)}{4\pi RC\alpha \int_0^L P_{ox}dx} + 1 \right].$$

If V_0 is negative and if the integral is positive, the right-hand side is greater than zero if $RC\alpha < 1$ and less than zero if $RC\alpha > 1$. In either case, τ_1 is positive, as is to be expected.

It is convenient at this stage to incorporate the expression (30) for V_m. Then

$$\frac{\tau_1}{RC}(1 - RC\alpha) = \log_e \left[1 - \frac{V_0}{V_m} \frac{(1 - RC\alpha)(RC\alpha)^{\frac{1}{1-RC\alpha}}}{RC\alpha}\right]. \tag{31}$$

In practice we shall find that $RC\alpha \ll 1$. Hence, very approximately

$$\tau_1 = RC \log_e \left[1 - \frac{V_0}{V_m}\right]. \tag{32}$$

In practice V_m is of the same order as $-V_0$, so that τ_1 is of the order 0.7 RC.

Three special cases

It will be observed that the maximum V_m is determined entirely by $RC\alpha$ for a given value of $\int_0^L P_{0x}dx$ and, of course, a given value of K. It is of interest to investigate three special cases as follows:

1. The case where $RC\alpha$ is large; that is, the case where R is large for a given α. It turns out that V_m has its greatest value when $R = \infty$ as might be surmised, and the value of V_m, which in this case we shall call $(V_m)_l$ is

$$(V_m)_l = \frac{4\pi}{K}\int_0^L P_{0x}dx \qquad \text{for } RC\alpha \text{ large.} \tag{33}$$

It is attained at $\tau_2 = \infty$.

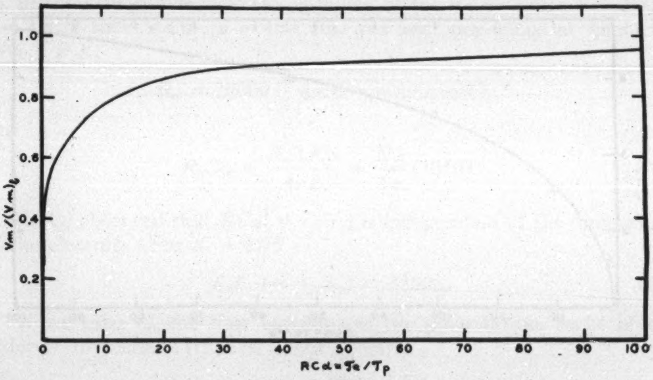

FIG. 3.

2. The case where $RC\alpha$ is small. Here it turns out that V_m, which in this case we shall call $(V_m)_s$, is given by

$$(V_m)_s = \frac{4\pi}{K}(RC\alpha)\int_0^L P_{0x}dx \qquad \text{for } RC\alpha \text{ small.} \tag{34}$$

The value is attained at

$$\tau_2 = -RC\log RC\alpha \qquad \text{approximately.} \tag{35}$$

3. The case where $RC\alpha = 1$; that is, where the time constant RC is equal to the time constant $1/\alpha$ associated with the decay. Here the value of V_m, which we shall call $(V_m)_M$ is given by

$$(V_m)_M = \frac{4\pi}{Ke}\int_0^L P_{0x}dx \qquad \text{for } RC\alpha = 1, \tag{36}$$

where e is the base of the Naperian system of logarithms. The value is attained at $\tau_2 = 1/\alpha$.

In order to avoid interrupting the general line of the discussion, we shall defer the proofs of these matters to an appendix to this paper (Problem 1).

Suppose $\tau_e(= RC)$ is the electrical time constant,
$\tau_p(= 1/\alpha)$ is the time constant of the polarization.

Then Fig. 3 shows $V_m/(V_m)_l$ for different values of τ_e/τ_p (that is, for different values of $RC\alpha$), plotted against τ_e/τ_p by use of (30).

It will be seen from (29) that if τ_2 is the time to attain any assigned maximum V_m, then τ_2/τ_p is a function only of τ_e/τ_p. Figure 4 shows τ_2/τ_p plotted against τ_e/τ_p.

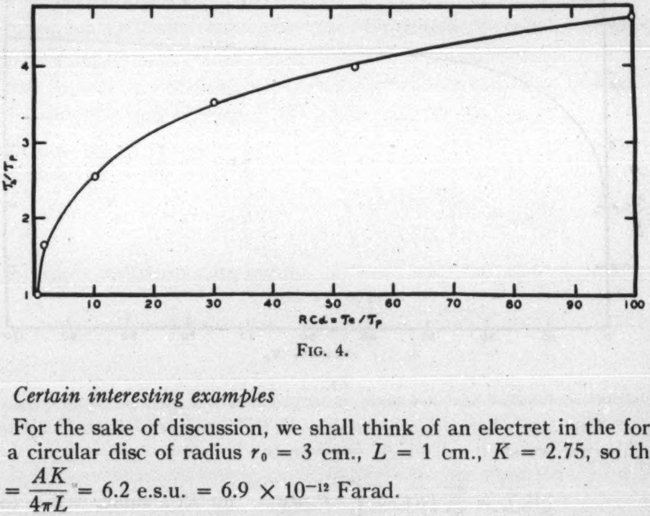

FIG. 4.

Certain interesting examples

For the sake of discussion, we shall think of an electret in the form of a circular disc of radius $r_0 = 3$ cm., $L = 1$ cm., $K = 2.75$, so that $C = \dfrac{AK}{4\pi L} = 6.2$ e.s.u. $= 6.9 \times 10^{-12}$ Farad.

It will be seen from Fig. 3 that $V_m/(V_m)_l$ attains 90 per cent of its greatest value (unity) for $\tau_e/\tau_p = 30$ and Fig. 4 shows that for this value of τ_e/τ_p, the quantity $\tau_2/\tau_p = 3.5$.

Let us consider an electret exposed to the free air, so that a discharging current comes from the atmosphere, let us say to the upper face, and let us suppose that this is the only leakage. Now the field will not be constant over the upper face. Indeed, to the extent that the system approximates to infinite parallel plates, the field will be zero except near the edges and for a finite electret it will vary over the surface. If the surface is bare, that is, without a foil or plate attached. the surface will not remain strictly at constant potential as the polarization decays, because a uniformly distributed layer of charge such as becomes effective as the decay proceeds would not result in constant potential over the surface to which it is attached. If, however, there is a foil on the surface, the potential will at least be kept constant over it,

For simplicity and in order to introduce definiteness into the problem, we shall suppose that there are foils on the surfaces.

In the appendix, Eq. 10*A*, it is shown that as regards atmospheric conductivity alone, the effective resistance between the two foils is approximately

$$R_a = (0.23Anev/L)^{-1},$$

where e is the electronic charge, n the number of ions (of one sign) per cc. in the atmosphere, v is the mobility of the ions. Since $n \sim 1000$; $e = 4.8 \times 10^{-10}$ e.s.u.; $v = 450$ cm. per sec. per e.s.u. of field, we have, for R_a

$$R_a = 20000 \frac{L}{A} \text{ e.s.u. approximately,}$$

and

$$R_a C\alpha = \frac{R_a A K\alpha}{4\pi L} = \frac{K\alpha}{4\pi}(1050).$$

It will be observed that $RC\alpha(= \tau_e/\tau_p)$ is independent of the dimensions of the electret. For $K = 2.75$

$$R_a C\alpha(= \tau_e/\tau_p) = 4200\alpha. \qquad (37)$$

If we take τ_p, the time constant of the polarization, to be of the order of 10^8 seconds (that is, about 3 years)

$$R_a C\alpha = 4200 \times 10^{-8} = 4.2 \times 10^{-5}$$

and is small, so that, from (29), the time τ_2 to reach the maximum will be

$$\tau_2 = -RC \log RC\alpha.$$

From (37) we see that

$$R_a C = 4200$$

so that

$$\tau_2 = -4200 \log_e (4.2 \times 10^{-5})$$

$$\tau_2 = 42{,}000 \text{ seconds}$$

and from (34) and (33)

$$(V_m)_e = \left[\frac{4\pi L}{K}\int_0^L P_{ox}dx\right][4.2 \times 10^{-5}] = 4.2 \times 10^{-5}(V_m)_l.$$

Thus $(V_m)_e$ only attains a very small fraction of the largest possible value given by (33) and attainable for $R = \infty$. However, this largest possible value $(V_m)_l$ may be very large indeed, as we shall see.

Now 42,000 seconds is small compared with the time for the observed apparent homocharge to attain its maximum. The observed value is about a week[11] from the time when V is zero.

For the case where $V_0 = -V_m$, Eq. 32 yields, for the time τ_1 necessary for V to change from V_0 to zero, $\tau_1 = 700$ seconds, which is small.

In practice the electret is not exposed to the full conductivity of the air, and the "saturation" current in the surrounding air may be, in general, small. If there is leakage in the dielectric, this leakage will determine the maximum value of V if the effective contribution by the air is negligible or if the electret is in a vacuum.[12] Suppose then we assign for R the value corresponding to the resistivity for good paraffin wax.[13] Then

$$R = \frac{L}{A} \times 3 \times 10^{18} \text{ ohms-cm.} = \frac{L}{A} \times 3 \times 10^{6} \text{ e.s.u., approximately.}$$

$$RC = \left(\frac{AK}{4\pi L}\right)\frac{L}{A} \times 3 \times 10^{6} = \frac{K}{4\pi} \times 3 \times 10^{6} = 6.6 \times 10^{5}.$$

Then, for $1/\alpha = 10^8$

$$\tau_2 = -RC \log RC\alpha = -6.6 \times 10^{5} \log_e \frac{6.6}{1000} = 3.3 \times 10^{6} \text{ sec.,}$$

which is now of the order of magnitude of the value found. The corresponding value of τ_1 is 4.6×10^5 sec. for the case where $V_0 = -V_m$.

It is probable that the specific resistance above assumed is high for ordinary waxes. If we should take 10^{18} ohms-cm., which is one-third of the value assumed above, we should find

$$\tau_2 = -RC \log RC\alpha = -2.2 \times 10^{5} \log_e 2.2 \times 10^{-3} = 13.4 \times 10^{5} \text{ sec.,}$$

which corresponds to about two weeks. The corresponding value of τ_1 is 1.5×10^5 sec. for $V_0 = -V_m$.

Then, from (33) and (34)

$$(V_m)_s = RC\alpha(V_m)_l = 2.2 \times (V_m)_l \times 10^{-3}. \qquad (38)$$

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[11] ANDREW GEMANT, "Electrets," *Physics Today*, Vol. 2, p. 8 (1949).

[12] Consider the case where we have the saturation current from a volume Al feeding the surface of the dielectric which is at potential V. At most, the number of ions produced per cc. per second would be less than 10, so that the saturation current i_s would be less than $10Ale$ e.s.u. The value of V/i_s is therefore greater than $(V/10Ale)$ (3×10^9) ohms if V is in volts. If $l = 2$ cm. and $V = 2000$ volts, this quantity is $6 \times 10^{20}/A$ which, for $L = 1$, is large compared with the value $L \times 3 \times 10^{18}/A$ cited for the internal resistance of the dielectric. Thus, in a confined space or in a vacuum, leakage through the air can in general be neglected compared with leakage resulting from internal resistance.

[13] G. W. C. KAYE AND T. H. LABY, "Tables of Physical and Chemical Constants and Some Mathematical Functions," London, Longmans Green, 9th edition, 1941, p. 93.

Under these conditions, and in view of (33), the electret, after attaining a potential difference of zero, would develop a potential which increased with the time to the value given by

$$(V_m)_s = (0.0022)\left(\frac{4\pi}{K}\right)\int_0^L P_{0x}dx, \tag{39}$$

which it would attain in about 14 days. After that it would decrease in accordance with (28) which, after a relatively short period, would become exponential in form and it would attain about one-third of its maximum value in about 3 years ($\sim 10^8$ sec.).

Taking the case where the initial semi-permanent polarization P_{0x} is constant through the dielectric and equal to P_0, it is of interest to inquire how large P_0 must be to cause $(V_m)_s$ to attain the value 7 e.s.u. ($= 2100$ volts).

We have, from (39), with this value of $(V_m)_s$,

$$7 = (2.2)\frac{4\pi}{K}P_0L \times 10^{-3}$$

so that, for $L = 1$ and $K = 2.75$

$$P_0 = 700 \text{ e.s.u.}$$

Now dipole moments of waxes are of the order 10^{-18} e.s.u. per molecule, and the number of molecules per cc. is of the order 10^{22} to 10^{23}. Thus, using 10^{23}, the maximum moment per unit volume would be $10^{23} \times 10^{-18} = 10^5$ e.s.u. Thus, the electret could achieve the foregoing conditions by becoming initially polarized to less than one per cent of its maximum polarization.

Note concerning the quantity of electricity which must flow to the plates in order to produce the polarization

If σ_0 is the quantity of electricity which goes to one of the plates during formation of the electret with a potential V_0 across the plates, we have

$$4\pi\sigma_0 = D = E + 4\pi P',$$

where P' is the total polarization.

In accordance with (1) therefore

$$4\pi\sigma_0 = E + (K - 1)E + 4\pi P_0 = KE + 4\pi P_0$$

$$\sigma_0 = \left(\frac{K}{4\pi}\right)\frac{\overline{V}_0}{L} + P_0.$$

If $\overline{V}_0 = 5000 \text{ volts} = 50/3 \text{ e.s.u.},$

if $P_0 = 700$ e.s.u., as above, and $L = 1$

$$\sigma_0 = 3.64 + 700 \sim 700 \text{ e.s.u.}$$

$$\sigma_0 = 2.3 \times 10^{-7} \text{ coulomb.}$$

This is a quantity readily measurable with a ballistic galvanometer. The success of measuring it in this manner would depend upon whether the *time* taken to produce the polarization was sufficiently short to insure that during that time the quantity of electricity conducted through the dielectric was inappreciable. Presumably it could only be inappreciable if the electret were formed at a temperature below the melting point.

The Field Outside an Electret

Some attention has been given to experimental measurements of the variation of the field with distance from the surface of the electret, and certain surprising results are quoted. Thus, F. Gutmann,[2] citing other observers, gives situations in which the field falls by a factor of 5 or 10 over a 5 mm. distance from the surface. In order to define the significance of the true field at a point outside the electret, one must consider carefully the dimensions of the electret. Apart from this, it is important to inquire the extent to which the measurements concerned do reveal the true field. Let us start by considering the first matter.

Insofar as the electret approximates to charged layers or polarized layers which are constant in planes of infinite extent, the field at finite distances from the surface must be zero if its potential is finite.[14] The existence of any finite external field at all depends therefore upon the finite size of the electret. In order to illustrate the essentials of this matter, it is convenient to think of the special case of two surface densities[15] σ and $-\sigma$, spread on two parallel discs of radius r_0 separated by a distance L.

It is, of course, obvious that if the plates are very near together the external field is of the same form as the magnetic field due to a magnetic shell and so is of the same form as the magnetic field due to a current flowing around the periphery of the shell.

In the case where the plates are separated, the field is of exactly the same form as the magnetic field of a solenoid of length L carrying a current of electricity or of a uniformly magnetized piece of material of length L and cross-sectional area πr_0^2. In either case, at great distances, the field would vary as the inverse cube of the distance.

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[14] If the total charge is not zero, the potential will be infinite for the case considered.

[15] These surface densities represent in each case the sum of the real and fictitious surface densities, so that no further allowance for the specific induction capacity of the dielectric has to become involved.

Of course, in the case of the existence of a volume density, our problem would take the form of one with an infinite number of discs.

It must be observed that the situation represented by two layers of constant surface density separated by a distance L is not exactly a system giving constant potentials over the surfaces. Indeed, a single disc at constant potential is very far from a surface of uniform surface density on each side. However, when two surfaces of equal and opposite surface density are very close together they do provide for constancy of potential over their area. Indeed, such a system is like a magnetic shell—representative of a current—and we recall immediately that the lines of magnetic force due to a current are perpendicular to the plane of the current as they pass through it.

If the z axis is taken perpendicular to the discs, if the origin is the center of the upper disc and if that disc is positive, we have, for the potential V_z along the axis above the upper plate

$$V_z = \sigma \int_0^{r_0} \frac{2\pi r dr}{(r^2 + z^2)^{\frac{1}{2}}} - \sigma \int_0^{r_0} \frac{2\pi r dr}{[r^2 + (z + L)^2]^{\frac{1}{2}}},$$

where r represents, of course, the radius of an annular ring of width dr on one of the discs. Thus

$$\begin{aligned} V_z &= 2\pi\sigma[\{r_0^2 + z^2\}^{\frac{1}{2}} - z - \{r_0^2 + (z + L)^2\}^{\frac{1}{2}} + z + L \\ &= 2\pi\sigma[\{r_0^2 + z^2\}^{\frac{1}{2}} - \{r_0^2 + (z + L)^2\}^{\frac{1}{2}} + L]. \end{aligned}$$

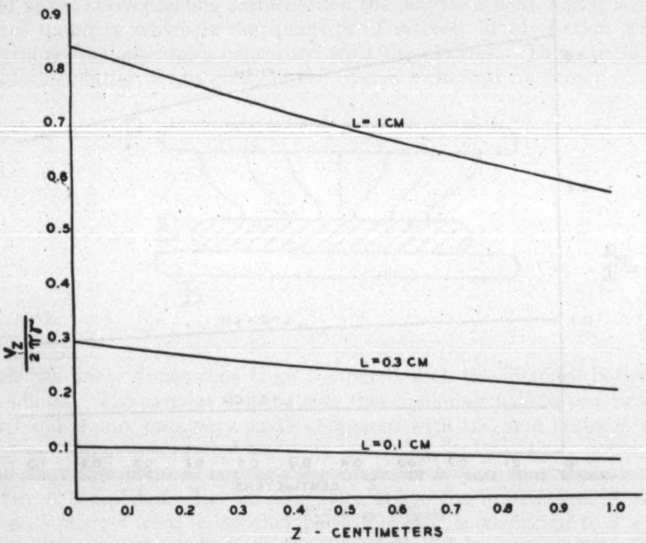

Fig. 5.

The field E_z is given by

$$E_z = 2\pi\sigma \left[\frac{(z + L)}{\{(z + L)^2 + r_0^2\}^{\frac{1}{2}}} - \frac{z}{\{z^2 + r_0^2\}^{\frac{1}{2}}} \right].$$

While it is possible to discuss the characteristics of V_z and E_z algebraically, it is more informative to represent the results graphically, and this is done in Figs. 5 and 6. It will be observed that the field does not show the sharp variation with distance from the surface cited by F. Gutmann in the example given above. In fact, in no case over the range of L cited (for $r_0 = 3$) does the field fall more than 10 per cent over the distance from the top surface to a point 0.5 cm. from that surface.

It may be recalled that the most general form of field outside of the dielectric may be defined as the field due to a suitably chosen moment per unit area, μ, over the surface combined with a suitably chosen charge per unit area, σ, over the surface.[16] By assigning very exceptional distribution μ and σ we can fit up almost any desired variation of the field with distance from the surface. However, before attempting such a procedure as a prelude to establishing significant conclusions concerning the nature of the charge distribution in the electret, it is desirable to inquire as to the nature of the measurements upon which the above-quoted variations of field with distance from the surface are based.

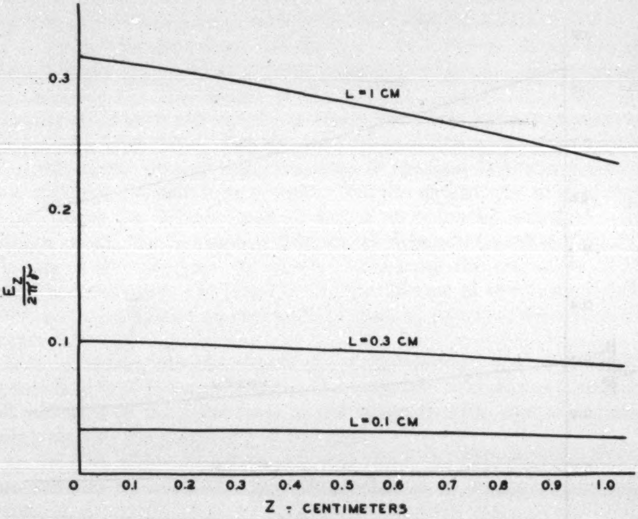

FIG. 6.

Referring to the paper by Andrew Gemant[17] from which the foregoing citations by F. Gutmann are made, we find that the procedure for measuring the field F, which was the quantity of interest, was as follows:

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[16] The theorem referred to is that of Green's equivalent stratum (see, for example, "Electricity and Magnetism," J. H. JEANS, third edition, pp. 182–183) in which the equivalent moment per unit area is shown to be $V/4\pi$ and that equivalent charge per unit area $-(\partial V/\partial n)/4\pi$, where V is the potential on the surface, and the differentiation is in the direction of the normal outwardly directed from the medium in which the field is sought.

[17] *Rev. Sci. Instr.*, Vol. 11, p. 65 (1940).

If, in Fig. 7, the shaded portion E represents the electret resting on a metal base at the zero of potential, P represents a metal plate attached to an electrometer system not shown. The experiment consists in measuring the charge Q on the plate P at various distances between P and B. It is then assumed that at any distance the field F is given by

$$F = 2\pi Q/f,$$

where f is the surface area of P.

Now, apart from minor considerations with which we shall not concern ourselves, it is true that F, as defined above, is the average field at the surface of the plate, but this field has no simple relation to the field at the corresponding surface when the plate is absent, and it is the latter quantity which is the quantity of interest in connection with anything fundamentally concerned with the electret. Thus, to illustrate this matter, let us consider the case of a charged condenser whose area has linear dimensions large compared with the distance between its plates. The average field outside this condenser will be practically zero and in any case very small compared with the field between the plates. For convenience, let us suppose the lower plate at zero potential and that the distance between the plates is h, and that there is air between the plates. Let us now place above the upper plate A, and at a distance h from it, another plate P. If P is connected to a very large capacity, the field between A and P will be equal to the field between the condenser plates and equal to half the field which existed between the condenser plates before P was introduced. The smaller the capacity connected to P the smaller the average field between A and P. However, in all cases the field between A and P has no fundamental relation to the field in the space above A when P is absent, which field is, in fact, as above stated, practically zero.

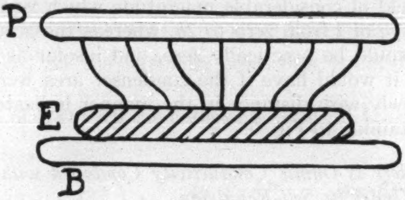

FIG. 7.

It is of interest to generalize the foregoing example as follows: Suppose that K is the specific inductive capacity of the material of the condenser, that σ_0 is the surface density on A before P is brought near, and F_{0c} is the field between the condenser plates under this condition. Suppose that when the plate is at a distance ϵ from A, the field between

P and A is F_p and the field between the condenser plates is F_c. Then, for the case where P is connected to a large capacity, we have as follows:

$$KF_{oc} = 4\pi\sigma_0$$
$$\epsilon F_p = hF_c$$
$$F_p + KF_c = 4\pi\sigma_0 = KF_{oc}.$$

Thus

$$F_p\left(1 + \frac{K\epsilon}{h}\right) = KF_{oc}$$
$$\frac{F_p}{F_{oc}} = \frac{Kh}{h + K\epsilon}.$$

Thus, if $K = 3$, the ratio of F_p for $\epsilon = 0$ to F_p for $\epsilon = 2h$ is $(h + 2hK)/h = 2K + 1 = 7$.

We can thus see how an experiment founded upon the foregoing principles could lead to the conclusion that there would be, outside the condenser, a field of considerable magnitude which varied by a factor of 7 for a change of ϵ from zero to $2h$, whereas the actual field in the absence of P would be practically zero, and insofar as it had a finite value, such as it would have if the condenser area were not large, it would vary slowly with distance in the manner indicated by the considerations pertaining to Fig. 6.

Case Where There is Ohmic Conductivity Combined with Motion of the Charge Under its Self-Repulsion

All of the previous developments and results apply up to and including Eq. 13. Only the time dependencies become affected by change in the law according to which the electricity disappears.

We shall generalize the expressions (14) for the current density to

$$i = \frac{d\sigma_1}{dt} + \lambda E_1 + \rho_1 u_1; \qquad i = -\frac{d\sigma_2}{dt} + \lambda E_2 + \rho_2 u_2, \tag{40}$$

where u is the velocity of the free electricity, so that

$$2i = \frac{d}{dt}(\sigma_1 - \sigma_2) + \lambda(E_1 + E_2) + \rho_1 u_1 + \rho_2 u_2.$$

From this point the procedure is exactly as in the case of Eqs. 16–19 inclusive, except that $2i$ is augmented by the term $\rho_1 u_1 + \rho_2 u_2$, with the result that (19) becomes augmented on the left-hand side by $\frac{2\pi L}{K}(\rho_1 u_1 + \rho_2 u_2)$.

Now, however, the equation of continuity becomes modified from (20) to

$$\frac{\partial \rho}{\partial t} + \lambda \operatorname{div} E + \operatorname{div} \rho u = 0$$

and (21) becomes modified to

$$\frac{\partial \rho}{\partial t} + \frac{4\pi\lambda}{K}\rho - \frac{4\pi\lambda}{K}\operatorname{div} P + \operatorname{div} \rho u = 0. \tag{41}$$

Equations 22 and 23 become modified to

$$\begin{aligned}\int_0^L \left(x\frac{\partial \rho}{\partial t} + \frac{4\pi\lambda x}{K}\rho \right) dx &= \frac{4\pi\lambda}{K}P_2 L - \frac{4\pi\lambda}{K}\int_0^L P dx - \int_0^L x\frac{\partial \rho u}{\partial x} dx \\ &= \frac{4\pi\lambda}{K}P_2 L - \frac{4\pi\lambda}{K}\int_0^L P dx - L\rho_2 u_2 + \int_0^L \rho u dx \end{aligned} \tag{42}$$

and

$$\int_0^L \left(\frac{\partial \rho}{\partial t} + \frac{4\pi\lambda}{K}\rho \right) dx = \frac{4\pi\lambda}{K}(P_2 - P_1) - (\rho_2 u_2 - \rho_1 u_1)$$

so that, bearing in mind that (19) becomes augmented, as aforesaid, by $\frac{2\pi L}{K}(\rho_1 u_1 + \rho_2 u_2)$ on the left-hand side, that equation, as modified, takes the form of (24) with the additional term $\frac{4\pi}{K}\int_0^L \rho u dx$ on the left-hand side. Thus

$$\frac{dV}{dt} + \left(\frac{1}{R_e C} + \frac{4\pi\lambda}{K} \right) V + \frac{4\pi}{K}\int_0^L \frac{\partial P}{\partial t} dx + \frac{4\pi}{K}\int_0^L \rho u dx = 0$$

and (25) becomes modified to

$$\frac{dV}{dt} + \frac{V}{RC} = -\frac{4\pi}{K}\int_0^L \frac{\partial P}{\partial t} dx - \frac{4\pi}{K}\int_0^L \rho u dx. \tag{43}$$

The two terms on the right-hand side, when multiplied by RC, act exactly like two external electromotive forces operating upon a circuit with resistance and capacity. Moreover, if we find a solution with one term alone giving $V = 0$ when $t = 0$, and if we find a solution with the other term alone giving $V = 0$ when $t = 0$, then the sum of these solutions will give a solution with the whole of the right-hand side of (43) operative, and that solution will make $V = 0$ when $t = 0$. In other words, the two contributions on the right-hand side of (43) act entirely independently except, of course, insofar as the fact that the term involving $\partial P/\partial t$ results in fields which control the value of ρu at each point. However, in terms of ρu regarded as known, the two terms in question act independently and it is convenient to consider them separately.

It will be observed that the term involving ρu, and arising from the motion of the electricity, acts exactly like a rate of change of polarization distribution both physically and formally. Physically a rate of change of polarization is representative of the motion of electricity of one sign relative to that of opposite sign, with a particular case that in which the electricity of opposite sign is fixed. Formally, we may define at each point an apparent polarization p such that

$$\frac{\partial p}{\partial t} = \rho u$$

$$(p_t)_x = (p_0)_x + \int_0^t \rho u dt,$$

where $(p_0)_x$ may be arbitrary since only time rates of change of $(p_t)_x$ are involved in our equations.

The actual calculation of ρu as a function of the time, in terms of an initial distribution, is a very involved problem, except in certain simple cases, and we shall defer consideration of special cases to a future publication, more particularly as it would appear that the most significant elements in the behavior of electrets can be covered by the considerations already given.

APPENDIX

Problem 1. Proof of Eqs. 33, 34, and 36.

Proof of (33) for the case RCα large

Put $x = RC\alpha$. Then we have to investigate the behavior of $y = x^{\overline{1-x}}$ with change of x. Put

$$z = \log y = \left(\frac{1}{1-x}\right) \log x.$$

Now from $x = 0$ to $x = 1$, the quantity $(1 - x)$ is positive, and $\log x$ is negative, decreasing in absolute value, and so increasing in actual value from $x = 0$ to $x = 1$. Since $1/(1 - x)$ also increases from $x = 0$ to $x = 1$, the quantity z, and so y, increases from $x = 0$ to $x = 1$. Now

$$\frac{dz}{dx} = \frac{1}{x}\left(\frac{1}{1-x}\right)\left[1 - \left(\frac{x}{x-1}\right) \log x\right].$$

Put $x - 1 = \epsilon$, so that

$$\frac{x}{x-1} \log x = \left(\frac{1+\epsilon}{\epsilon}\right) \log (1 + \epsilon) = \left(\frac{1+\epsilon}{\epsilon}\right) (\epsilon + \text{etc.}).$$

Hence, when $\epsilon = 0$, that is, when $x = 1$, $\left(\frac{x}{x-1}\right) \log x = 1$.

For $x > 1$, the quantity $\left(\frac{x}{x-1}\right) \log x$ increases continually with x.

Hence $\left[1 - \left(\frac{x}{x-1}\right) \log x\right]$ is negative and $\frac{1}{x}\left(\frac{1}{1-x}\right)\left[1 - \left(\frac{x}{x-1}\right) \log x\right]$

is positive for $x > 1$. Hence z, and consequently y, increases continually with increase of x from $x = 1$ to ∞, and therefore, in the light of what has already been proved, y increases continually from $x = 0$ to $x = \infty$.

Thus, y has its greatest value at $x = \infty$. Let us seek this value. At $x = \infty$,

$$z = \int_{\iota_{x=\infty}} \frac{\log x}{1 - x} = \int_{\iota_{\zeta=\infty}} \frac{\zeta}{1 - e^{\zeta}},$$

where $\zeta = \log x$.

Thus

$$z = -\int_{\iota_{\zeta=\infty}} \frac{\zeta}{e^{\zeta}} = 0.$$

Thus $y = 1$ for $x = \infty$; and (30) yields

$$(V_m)_l = \frac{4\pi}{K} \int_0^L P_{ox} dx.$$

From (29), we have

$$RC\alpha = e^{-\alpha\tau_2\left(-\frac{1}{RC\alpha}-1\right)} = e^{\alpha\tau_2} \qquad \text{for } RC\alpha \text{ large.}$$

Thus $\alpha\tau_2 = \log(RC\alpha) = \infty$ when $RC\alpha = \infty$.

Proof of (34) for the case $RC\alpha$ small

Referring to (30), we see that when $RC\alpha$ is small

$$(RC\alpha)^{\frac{1}{1-RC\alpha}} = RC\alpha$$

so that

$$(V_m)_s = \frac{4\pi}{K} RC\alpha \int_0^L P_{ox} dx.$$

From (29), we have

$$\begin{aligned} RC\alpha &= e^{-\frac{1}{RC}(1-RC\alpha)\tau_2} \\ &= e^{-\tau_2/RC} \qquad \text{for } RC\alpha \text{ small.} \end{aligned}$$

Hence

$$\tau_2 = -RC \log RC\alpha \qquad \text{for } RC\alpha \text{ small.}$$

Proof of (36) for the case $RC\alpha = 1$

For $RC\alpha = 1$, Eqs. 28, 29, and 30 become indeterminate. There is more than one way of proceeding. Perhaps the clearest is to go back to (28) and write it, for $V_0 = 0$,

$$V = \frac{4\pi RC\alpha e^{-\alpha t}}{(1 - RC\alpha)K} [1 - e^{-(1/RC-\alpha)t}] \int_0^L P_{ox} dx.$$

Then, when $RC\alpha$ is nearly equal to unity

$$e^{-(1/RC-\alpha)t} = 1 - \left(\frac{1}{RC} - \alpha\right)t$$

and

$$V = \frac{4\pi RC\alpha e^{-\alpha t}}{(1 - RC\alpha)K} \frac{t(1 - RC\alpha)}{RC} \int_0^L P_{ox}dx = \frac{4\pi\alpha}{K} te^{-\alpha t} \int_0^L P_{ox}dx.$$

This attains its maximum value for $t = \tau_2$ given by

$$\alpha\tau_2 = 1$$

and for this value

$$(V_m)_M = \frac{4\pi}{eK} \int_0^L P_{ox}dx.$$

Problem 2. The Effective Resistance of an Electret as Determined by the Conductivity of the Surrounding Air

By reference to footnote (3) and the paper cited in it, we see that if E_n is the normal component of the field at the upper plate of the electret,[18] the charge coming to the plate per second is J, where

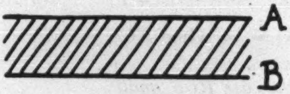

FIG. 8.

$$J = nev \int\int E_n ds, \tag{1-A}$$

where n is the number of ions per cc. in the free air, v is the mobility of the ions and e is the electronic charge. The field E_n is caused by the surface density (real and fictitious) on or near the surface of the upper plate, the surface density (real and fictitious) on or near the lower plate and any volume density (real and fictitious) in the dielectric itself.

For simplicity we shall confine attention to the case where there is no volume density. In this case, if σ and σ_f are the real and fictitious densities on the upper plate, those on the lower plate will be $-\sigma$ and $-\sigma_f$, respectively. We shall write

$$\sigma_0 = \sigma + \sigma_f. \tag{2-A}$$

If the plates are infinitely close together, they approximate the condition of an electric "shell," in analogy with a magnetic shell. Recalling that a magnetic shell is the equivalent of a current I of strength equal to the moment per unit area of the shell, we see that the field at the center of the top plate would, in this case, be given by E_0, where

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[18] The supposition is that the electret is in free air, in the sense that the lines of ionic flow to A can be calculated in such fashion that on tracing them backwards they do not meet obstacles which would initiate saturation.

$$E_0 = \frac{2\pi r_0}{r_0^2}\sigma_0 L,$$

where r_0 is the radius of the plate and L the infinitesimal distance between the plates. Of course, in this case, the potential difference between the plates would be V, where

$$V = 4\pi\sigma_0 L$$

so that, for a finite V, σ_0 would have to be infinite if L is infinitesimal. However, in terms of V, we should have

$$E_0 = \frac{V}{2r_0}.$$

We might thus expect $\int\int E_n ds$ to be of the order

$$\int\int E_n ds \sim \frac{V}{2r_0}(\pi r_0^2) \sim \frac{\pi}{2} r_0 V. \tag{3-A}$$

As a matter of fact, the real flux $\int\int E_n ds$ would be infinite in the case cited, the infinite contribution coming from a logarithmically infinite contribution near the boundary of the shell. This matter is, of course, representative of the fact that the self-induction of a loop of infinitely thin wire is logarithmically infinite. When the plates are not infinitely near together the infinity vanishes and the flux is of the order of magnitude of that given by (3-A).

While it is not really necessary to have discussed the limiting case of a shell, we have done so because it serves to make the whole picture more complete. The whole question of the flux is rather a subtle one since, for plates of infinite extent, the field for finite values of σ_0 is, of course, infinitesimal outside the plates.

Case where the plates are separated by a finite distance L

If

$$F \equiv \int\int E_n ds \tag{4-A}$$

and F_A is the contribution to this integral by the charge density σ_0 near to or on A, we have

$$F_A = (2\pi\sigma_0)\pi r_0^2. \tag{5-A}$$

The contribution to the flux F through A by the charge density $-\sigma_0$ near to or on B is not so easily found.

The potential U_z resulting from this charge at a point along the z axis with origin at 0 is readily seen to be

$$U_z = -\sigma_0 \int_0^{r_0} \frac{2\pi r dr}{(r^2 + z^2)^{\frac{1}{2}}} = -2\pi\sigma_0[(r_0^2 + z^2)^{\frac{1}{2}} - z].$$

We thus have

$$U_z = -2\pi\sigma_0 z\Bigg[1 + \frac{1}{2}\frac{r_0^2}{z^2} - \frac{1}{2^2\underline{|2}}\frac{r_0^4}{z^4} + \frac{1.3}{2^3\underline{|3}}\frac{r_0^6}{z^6} + \cdots$$
$$+ \cdots \frac{(-1)^{n+1}1\cdot3\cdot5\cdots(2n-3)}{2^n\underline{|n}}\frac{r_0^{2n}}{z^{2n}} + \cdots \text{etc.} - 1\Bigg]$$
$$\text{for } z > r_0. \qquad (6\text{-}A)$$

The value of U at any point P, with coordinates r, θ, is

$$U = -2\pi\sigma_0\Bigg[\frac{1}{2}\frac{r_0^2}{r}P_0(\mu) - \frac{1}{2^2\underline{|2}}\frac{r_0^4}{r^3}P_2(\mu) + \cdots$$
$$+ \cdots \frac{(-1)^{n+1}1\cdot3\cdots(2n-3)}{2^n\underline{|n}}\frac{r_0^{2n}}{r^{2n-1}}P_{2n-2}(\mu) + \cdots\Bigg]$$
$$\text{for } r > r_0, \qquad (7\text{-}A)$$

where the P's are the Legendre Polynomials, and $\mu = \cos\theta$.

Now the normal component of the flux over the plate is the same as the normal component of the flux over the spherical cap represented by the dotted line between M and N, Fig. 9. For all points on this cap, $r = \overline{ON} > r_0$. Hence (7-A) represents a satisfactory expression for U. We must take minus the derivative of U with respect to r, put $r = a$, where $a \equiv (L^2 + r_0^2)^{\frac{1}{2}}$, multiply by $2\pi a^2 \sin\theta d\theta$, that is, by $-2\pi a^2 d\mu$, and integrate from $\mu = 1$ to $\mu = \mu_1 = L/(L^2 + r_0^2)^{\frac{1}{2}}$.

On performing the foregoing operations, we find, for $(F_A)_B$, the flux through A due to the charges associated with B

$$(F_A)_B = \frac{(2\pi)^2}{2}\sigma_0 r_0^2\Bigg[\int_1^{\mu_1} P_0(\mu)d\mu - \frac{1\cdot3}{2\underline{|2}}\frac{r_0^2}{a^2}\int_1^{\mu_1} P_2(\mu)d\mu$$
$$+ \frac{1\cdot3\cdot5}{2^2\underline{|3}}\frac{r_0^4}{a^4}\int_1^{\mu_1} P_4(\mu)d\mu + \cdots \frac{(-1)^{n+1}1\cdot3\cdot5\cdots(2n-1)}{2^{n-1}\underline{|n}}$$
$$\times \frac{r_0^{2n-2}}{a^{2n-2}}\int_1^{\mu_1} P_{2n-2}(\mu)d\mu + \cdots\Bigg].$$

Now, if $m \neq 0$

$$m(m+1)P_m(\mu) = -\frac{d}{d\mu}\left[(1-\mu^2)\frac{dP_m(\mu)}{d\mu}\right]$$

$$\int_1^{\mu_1} P_m(\mu)d\mu = -\frac{(1-\mu_1^2)P'_m(\mu_1)}{m(m+1)} = -\frac{P'_m(\mu_1)}{m(m+1)}\sin^2\theta_1.$$

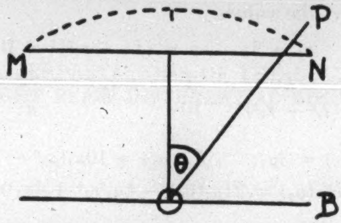

FIG. 9.

Thus, evaluating $\int_1^{\mu_1} P_0(\mu)d\mu$ as $(\mu_1 - 1)$

$$(F_A)_B = -2\pi^2\sigma_0 r_0^2 \sin^2\theta_1 \left[\frac{1-\mu_1}{\sin^2\theta_1} - \frac{1\cdot 3}{2\cdot 2\cdot 3\underline{|2}} \frac{r_0^2}{a^2} P'_2(\mu_1) \right.$$
$$+ \frac{1\cdot 3\cdot 5}{2^2\cdot 4\cdot 5\underline{|3}} \frac{r_0^4}{a^4} P'_4(\mu_1) - \cdots - \frac{1\cdot 3\cdot 5\cdot 7}{2^3\cdot 6\cdot 7\underline{|4}} \frac{r_0^6}{a^6} P'_6(\mu_1) + \cdots$$
$$\left. + \cdots \frac{(-1)^n 1\cdot 3\cdots(2n-1)}{2^{n-1}(2n-2)(2n-1)\underline{|n}} \frac{r_0^{2n-2}}{a^{2n-2}} P'_{2n-2}(\mu_1) \cdots \right].$$

Since

$$\sin^2\theta_1 = 1 - \mu_1^2 = \frac{r_0^2}{L^2 + r_0^2} = \frac{r_0^2}{a^2}$$

$$(F_A)_B = -\frac{2\pi^2\sigma_0 r_0^4}{a^2} \left[\frac{1}{1+\mu_1} - \frac{1}{2\cdot 2\underline{|2}} \frac{r_0^2}{a^2} P'_2(\mu_1) \right.$$
$$+ \frac{1\cdot 3}{2^2\cdot 4\underline{|3}} \frac{r_0^4}{a^4} P_4'(\mu_1) - \frac{1\cdot 3\cdot 5}{2^3\cdot 6\underline{|4}} \frac{r_0^6}{a^6} P'_6(\mu_1) + \cdots$$
$$\left. + (-1)^n \frac{1\cdot 3\cdot 5\cdots(2n-3)}{2^{n-1}(2n-2)\underline{|n}} \frac{r_0^{2n-2}}{a^{2n-2}} P'_{2n-2}(\mu_1) + \cdots \right]. \quad (8\text{-}A)$$

The value of F corresponding to (4-A) is now given by

$$F = F_A + (F_A)_B,$$

where F_A and $(F_A)_B$ are given by (5-A) and (8-A), respectively. Thus

$$F = 2\pi^2\sigma_0 r_0^2 \left[1 - \frac{r_0^2}{a^2(1+\mu_1)} \right]$$
$$+ \frac{\pi^2\sigma_0 r_0^6}{a^4} \left[\frac{P'_2(\mu_1)}{2\underline{|2}} - \frac{3}{2\cdot 4\underline{|3}} \frac{r_0^2}{a^2} P'_4(\mu_1) + \frac{1\cdot 3\cdot 5}{2^2\cdot 6\underline{|4}} \frac{r_0^4}{a^4} P'_6(\mu_1) + \cdots \right.$$
$$\left. + \cdots \frac{(-1)^n 1\cdot 3\cdot 5\cdots(2n-3)}{2^{n-1}(n-1)\underline{|n}} \frac{r_0^{2n-4}}{a^{2n-4}} P'_{2n-2}(\mu_1) + \cdots \right].$$

We shall now insert the values

$$L = 1; \qquad r_0 = 3; \qquad a = (L^2 + r_0^2)^{\frac{1}{2}} = 10^{\frac{1}{2}} = 3.16$$

$$\mu_1 = \frac{L}{(L^2 + r_0^2)^{\frac{1}{2}}} = \frac{1}{10^{\frac{1}{2}}} = 0.316; \qquad \frac{r_0^2}{a^2} = 0.9.$$

Now[19]

$$P'_2(\mu_1) = 3\mu_1; \qquad P'_4(\mu_1) = 10\mu_1(\mu_1^2 - \tfrac{3}{4}\nu_1^2);$$
$$P'_6(\mu_1) = 21\mu_1(\mu_1^4 - \tfrac{5}{2}\mu_1^2\nu_1^2 + \tfrac{5}{8}\nu_1^4),$$

where $\nu_1^2 = 1 - \mu_1^2$.

On inserting the above values, we find

$$F = 2\pi^2\sigma_0 r_0^2(0.47).$$

Now, if V is the potential difference between the plates

$$V = 4\pi(\sigma + \sigma_f)L = 4\pi\sigma_0 L.$$

Hence[20]

$$F = \iint E_n ds = \frac{\pi V}{2L} r_0^2(0.47). \qquad (9\text{-}A)$$

Hence, from (1-A) the charge coming to the upper plate per second is

$$J = \frac{(0.23)\pi r_0^2}{L} nevV$$

and the effective resistance R_a is

$$R_a = [(0.23)\pi r_0^2 nev/L]^{-1} = [(0.23)Anev/L]^{-1}$$

where A is the area of one of the foils.

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[19] J. CLERK MAXWELL, "A Treatise on Electricity and Magnetism," Vol. II, third edition, Oxford, Clarendon, 1892, p. 336.

[20] It is of interest to compare this with the very rough estimate $F \sim \pi r_0 V/2$ given by (3-A). Equation (9-A) yields, with $r_0 = 3$

$$F = (\pi r_0 V/2)(0.47 r_0/L) = 1.4(\pi r_0 V/2).$$

ADDITIONAL INTERPRETATIONS OF THE SOLUTION OF THE STRAIGHT BEAM DIFFERENTIAL EQUATION

BY

S. F. BORG[1]

INTRODUCTION

The fundamental differential equation of the transversely loaded straight beam, the Bernoulli-Euler equation, has been subject to various interpretive solutions. Perhaps the two best known solutions of this kind are the moment-area and conjugate beam methods which solve the equation with the aid of certain properties of the $\frac{M}{EI}$ curves for the beam in question.

The present paper applies two separate mathematical methods to the solution of this equation. The first method makes use of the Green's function and obtains a solution for the beam built-in both ends. The second method utilizes the so-called "superposition theorem" which is frequently applied to problems involving transient phenonema such as those encountered in electrical network and vibration problems, and obtains a solution valid for any type of support.

GREEN'S FUNCTION

Green's functions have been used to solve the classical vibrating beam problem (1).[2] In addition, Bateman (2) obtained Green's functions for the deflections of constant moment of inertia beams subject to various end conditions and pointed out the reciprocal theorem consequences and influence line applications that follow therefrom. More recently, Shaw (3), in solving the Fredholm integral equation illustrated a Green's function-relaxation solution for the simply supported beam-column. The present paper develops a solution for the beam of variable moment of inertia built-in both ends.

Given a non-homogeneous differential equation with boundary conditions (4)

$$\begin{cases} L(y) = r(x) \\ U_i(y) = 0, \ (i = 1,2, \cdots n) \end{cases} \tag{1}$$

then, if the Green's function for the homogeneous system

$$\begin{cases} L(u) = 0 \\ U_i(u) = 0, \ (i = 1,2, \cdots n) \end{cases} \tag{2}$$

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

exists and is known, the solution to system (1) is completely given by the integral equation

$$y(x) = \int_a^b G(x, \xi) r(\xi) d\xi,$$

in which $G(x, \xi)$ is the appropriate Green's function.

Thus, if we consider the simply supported beam, the system corresponding to (1) is

$$\begin{cases} \dfrac{d^2y}{dx^2} = \dfrac{M(x)}{EI(x)}, \\ y(0) = 0, \\ y(l) = 0. \end{cases} \tag{3}$$

The Green's function for this system is (5)

$$\begin{aligned} G(x, \xi) &= \frac{x}{l}(\xi - l) \qquad x \leq \xi \\ G(x, \xi) &= \frac{\xi}{l}(x - l) \qquad x \geq \xi \end{aligned} \tag{4}$$

and the solution is, therefore, given as

$$y(x) = \int_0^l G(x, \xi) \frac{M(\xi)}{EI(\xi)} d\xi. \tag{5}$$

Equation 5 will be recognized as the familiar Maxwell-Mohr unit dummy load equation

$$y(x) = \int_0^l m(x, \xi) \frac{M(\xi)}{EI(\xi)} d\xi,$$

in which $m(x, \xi)$ is the moment at ξ due to a 1-lb. load at x.

An interesting solution is obtained for the system

$$\begin{cases} \dfrac{d^2y}{dx^2} = \dfrac{M(x)}{EI(x)}, \\ y(0) = 0, \\ y'(l) = 0. \end{cases} \tag{6}$$

Collatz (6) lists the Green's function for this case as

$$\begin{aligned} G(x, \xi) &= x \qquad x \leq \xi \\ G(x, \xi) &= \xi \qquad x \geq \xi \end{aligned}$$

and consequently the solution is

$$y(x) = \int_0^x \xi \frac{M(\xi)}{EI(\xi)} d\xi + \int_x^l x \frac{M(\xi)}{EI(\xi)} d\xi.$$

We may interpret this result as follows in line with the Maxwell-Mohr solution: Given any beam with boundary conditions

$$y(0) = 0,$$
$$y'(l) = 0,$$

then applying the Maxwell-Mohr relation

$$y(x) = \int_0^l m(x, \xi) \frac{M(\xi)}{EI(\xi)} d\xi,$$

where m must now be obtained for the beam loaded and supported as shown in Fig. 1. Recalling that in the Maxwell-Mohr derivation only the unit load does external work in moving through the beam deflection, it is apparent that physical requirements of statics as well as the assumed work requirements are all satisfied by the given Green's function solution which is obtained almost entirely by mathematical reasoning.

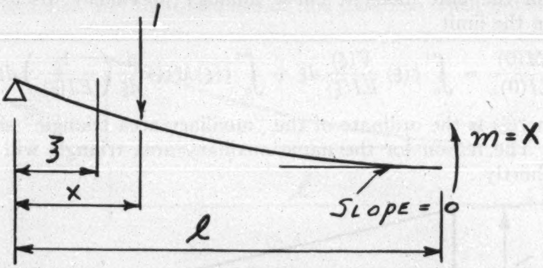

FIG. 1.

Another form of the beam differential equation is obtained by differentiating Eq. 3, and if the beam is built-in both ends we obtain

$$\begin{cases} \dfrac{d^2\theta}{dx^2} = \dfrac{d}{dx}\left(\dfrac{M(x)}{EI(x)}\right), \\ \theta(0) = 0, \\ \theta(l) = 0, \end{cases} \tag{7}$$

in which θ represents the slope of the neutral axis of the beam.

The Green's function for (7) is also given by (4), hence

$$\theta(x) = \int_0^l G(x, \xi) \frac{d}{d\xi}\left(\frac{M(\xi)}{EI(\xi)}\right) d\xi. \tag{8}$$

Equations 8 and 5 considered together give the following reciprocal relation: Given a beam A of constant section, built-in both ends and subjected to any transverse loading. Assume a second beam, B, of same length, simply supported. Then, if the $\frac{M}{EI}$ diagram of beam B is numerically equal to the $\frac{\text{Shear}}{EI}$ diagram of beam A, it follows that the deflection at any point on beam B is numerically equal to the slope at the same point on beam A.

Returning to Eq. 8, if we consider two sections of the beam a differential distance apart, we have

$$\theta(x + \Delta x) - \theta(x) = \int_0^l \{G(x + \Delta x, \xi) - G(x, \xi)\} \frac{d}{d\xi}\left(\frac{M(\xi)}{EI(\xi)}\right) d\xi. \quad (9)$$

Then, because

$$\frac{d\theta}{dx} = \frac{M}{EI}$$

$$\frac{dM}{dx} = V$$

and from the unit moment curve analogy previously discussed, we obtain in the limit

$$\frac{M(0)}{EI(0)_+} = \int_0^l \zeta(\xi) \frac{V(\xi)}{EI(\xi)} d\xi + \int_0^l \zeta(\xi) M(\xi) \frac{d}{d\xi}\left(\frac{1}{EI(\xi)}\right) d\xi, \quad (10)$$

in which $\zeta(\xi)$ is the ordinate of the "auxiliary area triangle" shown in Fig. 2. The reason for the name auxiliary area triangle will be discussed shortly.

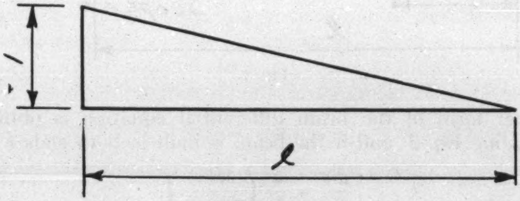

FIG. 2.

If the moment of inertia of the beam is constant, the last term in Eq. 10 disappears. If the moment of inertia varies in steps, Eq. 10 takes the form

$$\frac{M(0)}{EI(0)_+} = \int_0^l \zeta(\xi) \frac{V(\xi)}{EI(\xi)} d\xi + \Sigma\, \zeta(a) M(a) \left[\frac{1}{EI(a)_+} - \frac{1}{EI(a)_-}\right], \quad (11)$$

in which the summation is extended over all points, a, of step variation in moment of inertia, to the right of the left support.

Equation 11 is equivalent to

$$0 = \int_0^l \zeta(\xi) \frac{V(\xi)}{EI(\xi)} d\xi + \Sigma\, \zeta(a) M(a) \left[\frac{1}{EI(a)_+} - \frac{1}{EI(a)_-}\right], \quad (11a)$$

in which the summation term now includes all points of step variation in moment of inertia including the two supports, the walls being assumed of infinite moment of inertia. The signs of M and V follow the usual beam theory convention.

It may be verified that Eqs. 11 are equivalent to the first moment-area relation concerning the area of the $\frac{M}{EI}$ diagram. For this reason the auxiliary triangle is called the "auxiliary area triangle." Equations 11 are not sufficient to determine the two redundant quantities, M and V, at the support. To permit the calculation of both redundants, we note that the auxiliary area triangle is a form of operator such that its ordinate at any point, when multiplied by the appropriate shear, moment and stiffness values and summed over the entire beam, will give the area of the $\frac{M}{EI}$ curve. If these ordinates are integrated we obtain a parabola, and the ordinates of this parabola when multiplied by

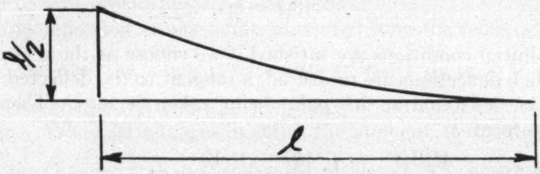

FIG. 3.

the same quantities will give an expression corresponding to the second moment area relation, that is, the moment about the right support of the $\frac{M}{EI}$ diagram.

Therefore, for the second independent relation, we have

$$0 = \int_0^l \eta(\xi) \frac{V(\xi)}{EI(\xi)} d\xi + \Sigma\, \eta(a) M(a) \left[\frac{1}{EI(a)_+} - \frac{1}{EI(a)_-} \right], \quad (12)$$

in which η is the ordinate of the "auxiliary moment parabola," Fig. 3, the other terms being as in Eq. 11.

THE SUPERPOSITION THEOREM

Given a constant coefficient differential equation

$$p(D)y = f \quad (13)$$

with

$$p(D) = C_0D^n + C_1D^{n-1} + \cdots C_n$$

then, following Murnaghan's (7) development we find

$$y(x) = f(0)_+ r_1(x) + \int_0^x r_1(x-\tau)Df(\tau)d\tau, \tag{14}$$

in which r_1 is the unit impulse "rest solution," that is, the solution of Eq. 13 corresponding to $f = 1$, and we must satisfy the initial conditions

$$y(0) = Dy(0) = \cdots = D^{n-1}y(0) = 0.$$

Equation 14 is the well known superposition theorem of electrical network and vibration theory, useful in the transient state solution.

Equation 14 is applied to the stationary beam problem as follows: Given

$$\frac{d^2y}{dx^2} = \frac{M(x)}{EI(x)}.$$

In this case $r_1(x)$ corresponds to the deflection caused by a distributed couple of value $EI(x)$ acting on a cantilever, hence

$$r_1(x) = \frac{x^2}{2}.$$

The initial conditions are satisfied if we choose as the reference line from which deflections are measured, a tangent to the deflected beam at $x = 0$, the deflection at this point being taken as zero. Then Eq. 14 takes the form

$$y(x) = \theta(0)x + \frac{M(0)x^2}{2EI(0)_+} + \int_0^x \frac{(x-\tau)^2}{2}\frac{V(\tau)}{EI(\tau)}d\tau + \int_0^x \frac{(x-\tau)^2}{2}M(\tau)D\left(\frac{1}{EI(\tau)}\right)d\tau, \tag{15}$$

$\theta(0)$ being the slope of the beam at $x = 0$.

Consider also

$$\frac{d\theta}{dx} = \frac{M(x)}{EI(x)}.$$

In this case $r_1(x)$ corresponds to the slope caused by a distributed couple of value $EI(x)$ acting on a cantilever, hence

$$r_1(x) = x.$$

The initial conditions are satisfied if we choose as the reference line, from which slopes are measured, a tangent to the deflected beam at $x = 0$. Then Eq. 15 takes the form

$$\theta(x) = \theta(0) + \frac{M(0)x}{EI(0)_+} + \int_0^x (x-\tau)\frac{V(\tau)}{EI(\tau)}d\tau + \int_0^x (x-\tau)M(\tau)D\left(\frac{1}{EI(\tau)}\right)d\tau. \tag{16}$$

If EI is constant the last terms of Eqs. 15 and 16 fall out. If EI varies in steps, Eqs. 15 and 16 take the form

$$y(x) = \theta(0)x + \int_0^x \frac{(x-\tau)^2}{2}\frac{V(\tau)}{EI(\tau)}d\tau + \sum \frac{(x-\alpha)^2}{2} M(\alpha)\left[\frac{1}{EI(\alpha)_+} - \frac{1}{EI(\alpha)_-}\right]. \quad (15a)$$

$$\theta(x) = \theta(0) + \int_0^x (x-\tau)\frac{V(\tau)}{EI(\tau)}d\tau + \sum (x-\alpha)M(\alpha)\left[\frac{1}{EI(\alpha)_+} - \frac{1}{EI(\alpha)_-}\right], \quad (16a)$$

in which α represents the distance from the left support to the points of step variation in EI, including the changes at the built-in ends, the walls in this case being assumed of infinite moment of inertia.

Equations 15 and 16 are somewhat more general than similar equations derived in a different manner by Compton and Dohrenwend (8) and may be interpreted physically as follows:

(*a*) The deflection at any point, measured from the reference line, is given by one-half the moment of inertia of the $\left\{\frac{V}{EI} + MD\left(\frac{1}{EI}\right)\right\}$ curves between the left end and the point, about the point of desired deflection. When EI changes in steps, the moment at the step, multiplied by the change in $\frac{1}{EI}$ at the step is considered as a concentrated $\frac{V}{EI}$ area.

(*b*) The slope at any point, measured from the reference tangent, is given by the moment of the $\left\{\frac{V}{EI} + MD\left(\frac{1}{EI}\right)\right\}$ curves between the left end and the point, about the point of desired slope. When EI varies in steps, the moment at the step is treated as in (*a*) above.

CONCLUSION

Two methods for solving the Bernoulli-Euler beam differential equation have been discussed. The first method used a Green's function and developed a solution for the beam built-in both ends in which the $\frac{V}{EI}$ and $MD\left(\frac{1}{EI}\right)$ diagrams, when operated on by an auxiliary area triangle and an auxiliary moment parabola gave expressions corresponding to the two moment area relations. The second method applied the superposition theorem to the problem and in this case the solution was obtained by taking appropriate moments of inertia and moments of the $\frac{V}{EI}$ and $MD\left(\frac{1}{EI}\right)$ curves.

REFERENCES

(1) **Philipp Frank und Richard v. Mises**, "Die Differential und Integralgleichungen der Mechanik und Physik," Braunschweig, Friedr. Vieweg & Sohn, 1930, p. 479.

(2) H. **Bateman**, "Partial Differential Equations of Mathematical Physics," New York, Dover Publications, 1944, pp. 16–20.

(3) F. S. **Shaw**, "The Approximate Numerical Solution of the Non-Homogeneous Linear Fredholm Integral Equation by Relaxation Methods," *Quarterly of Applied Mathematics*, Vol. 6, No. 1, April, 1948, p. 69.

(4) E. L. **Ince**, "Ordinary Differential Equations," New York, Dover Publications, 1944, p. 256.

(5) *Loc. cit.*, p. 258.

(6) **Lothar Collatz**, "Eigenwert Probleme," Leipzig, Becker und Erler, 1945, p. 83.

(7) **Francis D. Murnaghan**, "Introduction to Applied Mathematics," New York, John Wiley & Sons, 1948, p. 354.

(8) H. B. **Compton and** C. O. **Dohrenwend**, "The Shear Area Method," *Transactions American Society of Civil Engineers*, Vol. 101, 1936, p. 945.

NOTES FROM THE NATIONAL BUREAU OF STANDARDS*

SEAC—THE NATIONAL BUREAU OF STANDARDS EASTERN AUTOMATIC COMPUTER

The completion and successful operation of SEAC—the National Bureau of Standards Eastern Automatic Computer—has been achieved by electronic scientists of the National Bureau of Standards.

SEAC is a high-speed, general-purpose, automatically-sequenced electronic computer. It was developed and constructed, in a period of 20 months, by the staff of the National Bureau of Standards under the sponsorship of the Department of the Air Force to provide a high-speed computing service for Air Force Project SCOOP (Scientific Computation of Optimum Programs), a pioneering effort in the application of scientific principles to the large-scale problems of military management and administration. SEAC will also be available for solving important NBS problems of general scientific and engineering interest.

SEAC automatically performs all of the logical and arithmetical operations required to solve a particular problem when it is supplied with coded instructions and numerical data. By combining a vast number of simple operations into a complex, high-speed sequence, it can calculate the answers to many difficult computational and statistical problems whose solutions otherwise would be impractical in any reasonable time or energy sense.

All operations are carried on in a binary number system, the presence of a pulse indicating "1" and the absence of a pulse indicating "0." In SEAC a sequence of 45 binary digits is known as a "word" and may convey operational instructions as well as numerical data. SEAC's high computing speed is largely the result of two design features: its rapid pulse rate (1 megacycle per second) and its large memory (512 words) capacity. They make it possible for SEAC to add or subtract pairs of 11-digit numbers 1100 times a second and multiply or divide them 330 times a second. These rates include the time it takes for the machine to search its memory for the numbers, operate upon them and return the result to the memory. An arithmetical operation of addition or subtraction alone is completed in 50 microseconds; multiplication or division is completed in 2500 microseconds.

SEAC's computing functions are carried on in four main sections: an input-output unit, a memory unit, a control unit, and an arithmetic unit. The input-output unit is the link between the machine and its operators. Here information and instructions are fed in and answers are printed out.

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* Communicated by the Director.

The memory unit acts as a file for instructions and numbers while the control unit is directing the flow of information through the computer. Signals from the control unit perform the electronic switching which sends numbers and instructions to the right places at the right times. The control unit continually refers to the memory unit for instructions in the proper sequence, and the arithmetic unit carries out the actual computations the control unit directs it to perform. When the arithmetic unit has arrived at the solution required, the control unit directs the memory unit to send the answer to the output unit.

The present input-output unit employs a manual keyboard for direct input and a teletype printer for direct output, using a hexadecimal notation (base 16) to represent numbers and instructions. Indirect operation is accomplished through the use of punched paper tape. SEAC's design allows for the later replacement of the teletype system by a faster magnetic wire system. The initial model of a magnetic wire input-output unit with a single serial information channel has already been planned. The time to read a block of 8 words in or out with the magnetic wire scheme is less than a tenth of a second; with the present punched tape it is 16 seconds. SEAC is flexible enough to accommodate a wide range of serial or parallel input-output systems, so that future developments can be directly incorporated.

The main memory now operating in the SEAC is a serial unit with a storage capacity of 512 words (8 words in each of 64 acoustic delay lines). A word is stored as a series of sound waves travelling in mercury: waves generated by a quartz crystal at one end of a mercury-filled tube are received at the other by a similar crystal. A special amplifier then transforms the sound energy into electrical signals and returns the pulse sequence, at its original strength, to the first crystal. In this way a word is recirculated until it is called for by the control unit. Average access time in referring to a word in the memory is 168 microseconds. Plans for future expansion call for the addition of a parallel memory system of 45 electrostatic tubes storing an additional 512 or 1024 words with an access time of only 12 microseconds.

The preparation of an instruction program for a problem to be solved on SEAC requires a great deal of skilled work by specially trained mathematicians. But once the program for a particular kind of problem has been coded, it can be used over and over again with different sets of numerical data. The changeover from one type of problem to another is accomplished by sending in a new set of instructions through the input-output unit. No rewiring or switch-setting is required. Instructions previously read into the machine may also be altered without complete erasure. This simplifies the preparation of problems since less complete instructions are needed initially.

SEAC has already solved an important problem in optics in the design of lenses and one in thermodynamics; and several mathematical problems of general interest to pure mathematicians were solved earlier for the purpose of testing the machine. In one of these the machine was directed to compute the factors of any given number up to 100 billion. It rapidly determined, for example, that the number 99,999,999,977 has no factors and is therefore a prime number. In order to do this, the machine divided 99,999,999,977 by 80,000 different trial divisors. SEAC generated each of these trial devisors, automatically establishing that each was a prime number. This problem, which would be two months' work for a man operating a desk calculator 8 hours a day, required 30 minutes for its solution.

In the development of SEAC, emphasis was placed on designing circuits especially for computer use rather than adopting the standard procedures of television and radar circuitry. All computing and switching in SEAC is performed by germanium crystal diodes rather than by electron tubes; tubes are used only for power amplification. Standardized tube-and-transformer combinations are used throughout the machine to simplify maintenance. The transformer method provides high-frequency coupling with a minimum of cross-talk between circuits. These advances in circuit design as well as several unique construction features make it easy to add units to the machine, and the present version of SEAC could serve as the nucleus of a much larger computer.

Because SEAC was needed for use as quickly as possible, its design was kept simple, and the equipment which had to be built was held to a minimum. SEAC was designed to operate initially as a serial machine because this mode of operation requires less equipment than the parallel method; this does not, however, mean less functional effectiveness. The list of basic operations or orders was kept as short as possible for the same reason. SEAC has seven basic orders, chosen after a careful study which established their convenience for solving the most elaborate problems the machine is expected to handle: addition, subtraction, multiplication, division, comparison, logical transfer, and input-output control. The comparison order allows two numbers to be compared to determine which is larger. This feature gives the machine a degree of choice. The result of a comparison thus dictates which of two alternative operational sequences the machine is to perform next. The logical transfer order makes it possible for the machine to alter an instruction it has previously received.

All of these specific SEAC design considerations are a part of a much broader NBS program on automatic electronic computers. The program has four principal phases: fundamental research, engineering development, machine design and construction, and technical coordination. The fundamental research includes basic studies in numerical analysis, logical design, programming techniques, and circuit efficiency. The long-range objective of this research is faster, simpler, and more versatile computers, and more effective use of the machines already in existence.

In the NBS engineering development work theoretical design ideas are translated into actual models. In this way, the relative advantages and limitations of different kinds of electrical and mechanical apparatus become apparent. Developmental work has been concentrated on certain critical computer needs: efficient memory systems, high-speed switching, and increased reliability of electronic components.

The design and construction phase represents the application of the knowledge derived from NBS research and development work and from current advances in the computer field. In addition to SEAC, the National Bureau of Standards has undertaken the actual construction of a second machine, SWAC, which is rapidly approaching completion in the NBS Los Angeles laboratories. The computer design and construction projects have already provided valuable engineering training and experience and will make possible extensive programming and operating practice.

The final phase of the NBS computer program is in providing technical services for other Government agencies. This involves, in addition to mathematical and electronic consulting services and the construction and operation of SEAC and SWAC, technical coordination of contracts placed with industry for five other large-scale computers: one for the Bureau of the Census to tabulate and compute statistical information, one for the Air Force Comptroller for further work on program planning problems, one for the Army Map Service for calculations arising in map adjustment, one for the Office of Air Research to handle engineering computations, and one to be operated by the NBS Computation Laboratory for the Office of Naval Research.

LOW-TEMPERATURE POTENTIALITIES OF THE SILICONE RUBBERS

A recent investigation at the National Bureau of Standards under the joint sponsorship of the Office of Naval Research and the Quartermaster Corps has shown that the silicone rubbers, developed especially for high-temperature applications, have better potentialities for use at extremely low temperatures than any synthetic or natural rubber studied thus far. This conclusion is based primarily on measurements[1] of the second-order transition temperature of the silicone rubbers by C. E. Weir, W. H. Leser, and L. A. Wood of the Bureau staff.

The silicones are synthetic rubbers in which some of the carbon atoms normally present are replaced by silicon and oxygen. Since these materials are highly resistant to heat, retaining their elasticity and electrical resistance at temperatures as high as 200° C. (400° F.), they have been found especially well suited for hose and gaskets in airplane engines and for insulated cables. In recent years a need has also arisen for a type of rubber capable of withstanding low temperatures without loss of its characteristic rubber-like properties. Tires, belting, or other articles of ordinary rubber lose their elasticity around −50° C. (−60° F.), presenting many difficulties in connection with the operation of motor vehicles and machinery in the arctic or of airplanes at great height. To learn more about the possibilities of using the silicones for such low-temperature applications, the National Bureau of Standards undertook an investigation to determine the lower limit of the temperature range in which they retain their characteristic elasticity.

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[1] For further technical details, see "Crystallization and Second-Order Transitions in Silicone Rubbers," by C. E. Weir, W. H. Leser, and L. A. Wood, *J. Research, NBS*, Vol. 44, p. 367 (1950) RP2084.

This lower limit was determined by locating the second-order transition temperature, a temperature at which a marked change in the slope of the length-temperature curve occurs. Such a change is observed in all rubbers and plastics and can be recognized as a discontinuity in the derivatives of volume, heat content, index of refraction, compressibility, dielectric constant, and other quantities with respect to temperature. A second-order transition differs from the ordinary first-order transition, or change of phase, in that no volume change or latent heat is involved. However, below the second-order transition temperature the type of molecular motion responsible for the useful properties of a rubber ceases, and the material behaves essentially as an ordinary solid. In practice a rubber becomes useless for applications requiring long-range elasticity at temperatures somewhat higher than the transition temperature, the exact amount of the difference depending on the particular application. Thus in natural rubber the second-order transition temperature is at −70° C. (−94° F.), but the rubber is seldom useful below about −55° C. (−67° F.).

In the Bureau's investigation, the necessary thermal expansion measurements were made in an interferometer. Small slabs of rubber were placed between the two quartz interferometer plates. Then, as the temperature of the interferometer was changed, the rubber slabs contracted or expanded, changing the distance between the plates and causing the interference fringes to the eyepiece to move relative to a fixed point of reference. The temperature of the specimens was recorded, and the number of fringes passing the reference point was counted as the temperature was varied slowly from −196° C. (−321° F.), the boiling point of liquid nitrogen to 100° C. (212° F.)

The silicone rubbers studied were all of commercial origin, and all except two, which were pure-gum silicones, contained fillers and vulcanizing agents. Several of the samples were especially designed for low-temperature applications. As only small differences in transition temperature were observed among any of these samples, it was concluded that fillers and vulcanizing agents have little effect on the second-order transition temperature. Additional measurements on two commercial silicone rubbers not designed for low temperatures were in substantial agreement with those for the low-temperature silicones.

One variety of low-temperature silicone rubber was outstanding in that its expansion curve was essentially linear from 100° C. (212° F.) until the second-order transition temperature of approximately −123° C. (−189° F.) was reached. Below this temperature a much lower coefficient of thermal expansion, more nearly like that of a rigid solid, was observed. All other silicone rubbers studied likewise exhibited a second-order transition at approximately −123° C., the lowest temperature at which such a transition has been observed in a polymeric material. However, the other silicones, on being cooled from room temperature, also went through a first-order transition at temperatures varying from −65° C. (−85° F.) to −75° C. (−103° F.).

The first-order transition was observed experimentally as the very rapid passage of a large number of fringes past the reference point while the temperature was varied over a narrow range. This effect was interpreted as corresponding to a considerable decrease in volume associated with partial crystallization of the amorphous silicone. The crystallization process produces some stiffening and has apparently prevented the successful use of silicone rubbers below this temperature. However, the one variety which exhibited no first-order transition should have good possibilities for use at temperatures as low as −100° C. (−148° F.).

On heating the partially crystalline material, melting was found to occur over a range of temperatures considerably higher than those required to produce crystallization. This behavior is analogous to that shown by natural rubber. The volume change of the pure-gum silicone rubber on crystallization or melting varied over a range from 2.0 to 7.8 per cent. Between 0° C. and −35° C., the expansivity of the pure material was about 40 × 10^{-5}/deg. C. Similar data were obtained for the silicones containing fillers and vulcanizing agents, but the values obtained were lower because of the presence of these ingredients.

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RECENT ADDITIONS

AVIATION

CHAPEL, CHARLES EDWARD. Jet Aircraft Simplified. 1950.

BIBLIOGRAPHY

GIBSON, R. W. Francis Bacon; a Bibliography of His Works and of Baconiana. 1950.

BIOGRAPHY

ANDRADE, EDWARD NEVILLE DA COSTA. Isaac Newton. 1950.

COULSON, THOMAS. Joseph Henry; His Life and Work. 1950.

LODGE, SIR JOSEPH. Past Years; an Autobiography. 1932.

MIGNET, M. Vita di Franklin ad Uso di Tutti. n.d.

CIVIL ENGINEERING

TORRINGTON COMPANY, BANTAM BEARINGS DIVISION. Anti-friction Bearing Designs for Movable Span Bridges. 1950.

ELECTRICITY AND ELECTRICAL ENGINEERING

GRABNER, ALFRED. Elektridynamische Starkstrammaschinen. 1944.

MILLER, HENRY A. Cold Cathode Fluorescent Lighting. 1949.

ENGINEERING

BARBER, THOMAS WALTER. The Engineer's Sketch-book. 1948.
BOYD, JAMES ELLSWORTH. Strength of Materials. Edition 5. 1950.

GENERAL

VOGEL, WILLIAM P., JR. Precision, People and Progress; a Business Philosophy. 1949.

MANUFACTURE

BRINKLEY, JOHN. Design for Printing; a Handbook of Design and Reproduction Processes. 1950.
HAYTER, STANLEY WILLIAM. New Ways of Gravure. 1949.
MICHELON, LENO C. Industrial Inspection Methods. 1950.
SORRELL, S. E. Paper Base Laminates. 1950.
VERSTONE, P. E. The Manufacture of Paper Containers. 1949.

MARINE ENGINEERING

DIESEL ENGINE MANUFACTURERS ASSOCIATION. Marine Diesel Standard Practice. Edition 2. 1948.

MATHEMATICS

ZWIKKER, C. Advanced Plane Geometry. 1950.

MECHANICAL ENGINEERING

GRAMBERG, A. Maschinenunter Suchungen und das Verhalten der Maschinen im Betriebe ein Handbuch. 1921.
SALISBURY, J. KENNETH. Steam Turbines and Their Cycles. 1950.

METALLURGY

MILLER, RICHARD FRANKLIN AND HEGER, JAMES J. Report on Strength of Wrought Steels at Elevated Temperatures. 1950.
ROBESONIA IRON COMPANY. One Hundred and Twenty Five Years of Pig Iron Manufacture at Robesonia, Pennsylvania. 1918.
SMITH, GEORGE V. Properties of Metals at Elevated Temperatures. 1950.

MILITARY ENGINEERING

CRAIGE, JOHN HOUSTON. The Practical Book of American Guns. 1950.

PETROLEUM

GEORGI, CARL W. Motor Oils and Engine Lubrication. 1950.
LUMMUS COMPANY. Petroleum Horizons. 1950.

PHYSICS

BOSE, SIR JAGADIS CHUNDER. Collected Physical Papers. 1927.
EINSTEIN, ALBERT. Über die Spezielle und die Allgemeine Relativitäts Theorie. 1921.
FINKELNBURG, WOLFGANG. Atomic Physics. 1950.
GAYNOR, FRANK, Ed. Pocket Encyclopedia of Atomic Energy. 1950.
GEMANT, ANDREW. Frictional Phenomena. 1950.
HOUSNER, GEORGE WILLIAM AND HUDSON, DONALD ELLIS. Applied Mechanics Dynamics. 1950.
INSTITUTE OF PHYSICS. The Acceleration of Particles to High Energies. 1950.
MATZ, WERNER. Die Thermodynamik des Wärme und Stoffaustches in der Verfahrenstechnik. 1949.
SOMMERFELD, ARNOLD. Mechanics of Deformable Bodies. 1950.
SPREADBURY, F. G. Permanent Magnets. 1949.
TESZNER, STANISLAS. Semi-conducteurs Électroniques et Complexes Dérives. 1950.

RAILROADS

Allen, Cecil J. Locomotive Practice and Performances in the 20th Century. 1949.

SCIENCE

Bavink, Bernhard. The Natural Sciences. 1932.
Standen, Anthony. Science is a Sacred Cow. 1950.

SUGAR

Emmen, Q. A. D. Rietsuiker Fabrieken Op. 1926.

MUSEUM

Every boy who has seen a steam locomotive panting in a railroad station has formed a desire to be an engine driver. The Institute Museum has taken account of the suppressed desires resulting from the frustrations arising from a failure to accomplish the wish to drive a locomotive, by providing the means by which boys of all ages can realize an ambition. A giant locomotive is one of the exhibits into which the visitor may joyfully clamber, push over the throttle control, and feel the huge mechanism respond to the action. The locomotive moves! It does not develop any high speed, it is true, but it does move up and down the track in response to the controls operated by the visitor.

After having driven the locomotive the visitor usually descends, looks up at the complicated mass of mechanism, and begins to ask questions at a rate which not infrequently baffles the guide. Most locomotives are built to fill a routine need, but this one, #60,000, has a very special reason for existing. It was built as an experiment to ascertain the economies that might be effected by the use of high pressure steam and a high ratio of expansion. The experiment was judged necessary by the tendency of locomotives to expand in size and weight in proportion to the increase in horse power. Growth in size led to a wish to develop an engine which would furnish equal or greater horse power within a smaller compass.

It should be observed that locomotive engineers have definite limits for steam temperatures between which they must work, and the wish to take advantage of greater expansion was necessarily accompanied by an obligation to increase cylinder size, which was undesirable. The conventional steam pressure in locomotives at the time #60,000 was built was 200 to 215 lb. to the square inch. This locomotive was designed in 1926 to give greater economy with steam at 350 lb. to the square inch. This involved the use of a partial water-tube boiler in place of the usual fire-tube boiler, and the use of three cylinders compounded. Steam was first admitted to a middle cylinder and, after expansion there, passed to two low pressure cylinders where additional expansion took place. The cylinders are uniform in size, 27 by 32 inches. The indicated horse power developed in tests was 4515.

The boiler is 84 inches in diameter. The driving wheels are 63½ inches in diameter, and the total weight of the engine alone is 457,000 pounds.

An invariable question is asked by visitors: How did we get the engine into the building? The end of the building was omitted to admit the entrance of the locomotive, which was drawn down Twenty-first street by trucks, and pushed into place. But few people ever think of the more crucial problem. What kind of floor does the great locomotive stand on? Obviously, this can be no ordinary floor to support such a weight. Unfortunately, visitors are not now admitted to the room which holds the secret of this support, otherwise they would be able to see that the locomotive runs on a track supported on two small bridges normally used for carrying track over creeks. These bridges were the gift of the Pennsylvania Railroad Company.

The locomotive itself was built by the Baldwin Locomotive Works who presented it to the Museum after it had run its experimental tests on various types of track.

THE FRANKLIN INSTITUTE LABORATORIES FOR RESEARCH AND DEVELOPMENT

Abstract of **Factors Affecting the Precision of Viscosity Measurements with the Torsion Crystal.**[1]—P. E. ROUSE, JR.,[2] E. D. BAILEY,[2] and JEAN A. MINKIN.[2] This paper gives the results of a study whose objective was the refinement of a technique which promises to be of great value in the study of complex liquids. As a result of this work it is now possible to measure the high frequency viscosity of liquids with almost as high a precision as can be attained with steady-state flow instruments.

The torsion crystal method of studying the complex high frequency shear reaction of liquids was introduced by W. P. Mason[3] of the Bell Telephone Laboratories in 1946. This method makes it possible to measure the component of the liquid reaction in phase with the velocity of the crystal surface and the component 90° out of phase with this velocity. When the high frequency reaction of a liquid can be entirely described by the steady-flow viscosity and density of the liquid, the magnitudes of the in-phase and out-phase components of the high frequency reaction are equal. In the case of liquid polymers, polymer solutions and other multi-component systems the component in phase with the velocity of surface is larger than the out-phase at frequencies which can be attained experimentally. A knowledge of the values of the components of the reaction as a function of frequency over a wide frequency range makes it possible to draw important conclusions concerning the structure of a liquid system.

The usefulness of the information which can be supplied by a torsion crystal study is very dependent on the precision with which the measurements can be carried out. This is a consequence of the fact that it is essential to know when the in-phase and out-of-phase components of the liquid reaction do differ and to know the magnitude of their differences. The study of very dilute polymer solution is an important application of the method, and since the reaction of such systems differs very little from the pure solvent, the measured values must be known very accurately if the contribution of the polymer to the total reaction is to be studied. It is equally important that the behavior of the crystal be very reproducible.

In this technique a cylindrical crystal of quartz is oscillated in a liquid medium. The resonant amplitude, which is inversely proportional to the in-phase component of the reaction, and the resonant frequency, which changes linearly with the out-phase component, are determined. Since the crystal system may be represented as a series resonant circuit (Fig. 1), the resonant amplitude is determined from the value of the resonant resistance measured with a conductance-capacitance budge. The resonant frequency is measured with an electronic counter used in conjunction with a precision tuning fork. The resonant resistance can be measured to 0.2 per cent for liquids whose viscosities fall in the range from 1 centipoise to 1 poise. For liquids in this viscosity range the resonant frequency can be measured to one part in 200,000. This means that the uncertainty in the change in resonant frequency produced by the reaction of the liquid is 4 per cent when the liquid viscosity is 0.5 centipoise and 0.2 per cent when the liquid viscosity is one poise. These values compare with a precision of 10–15 per cent reported in early work.[3] Figure 2 shows graphically the agreement between the measured values found in the recent study and the theoretical relationships derived by Mason. This figure illustrates the difference in behavior of a simple liquid and a liquid in which the reaction depends on processes whose relaxation times are of the same order as the period of oscillation of the crystal. The linear plots shown were obtained from the measured reaction of solvents and low molecular

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[1] Paper presented at a Symposium on Analytical Research during the 15th Mid-Year Meeting of the American Petroleum Institute's Division of Refining, in the Hotel Cleveland, Cleveland, Ohio, May 1, 1950. The work reported herein was initiated under a grant from the American Petroleum Institute. It has been continued under sponsorship of the Office of Rubber Reserve, Reconstruction Finance Corp., in connection with the government's synthetic rubber program.

[2] Physicist, Research Engineer, and Research Assistant, respectively, The Franklin Institute Laboratories for Research and Development, Philadelphia 3, Pa.

[3] W. P. MASON, *Trans. ASME*, Vol. 69, p. 359 (1947).

NOTE: Reprints of this paper are available upon request to the Administration Division, The Franklin Institute Laboratories for Research and Development, Parkway at 20th St., Philadelphia 3, Pa.

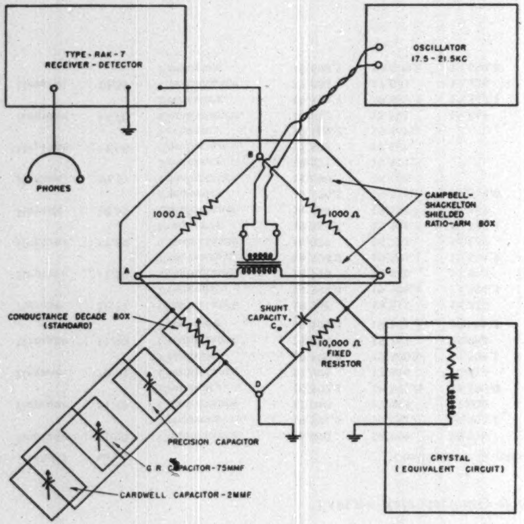
TYPE - RAK - 7
RECEIVER - DETECTOR
PHONES
OSCILLATOR
17.5 - 21.5KC
B
1000 Ω
1000 Ω
CAMPBELL-
SHACKELTON
SHIELDED
RATIO-ARM BOX
A
C
SHUNT
CAPACITY,
C_0
10,000 Ω
FIXED
RESISTOR
CONDUCTANCE DECADE BOX
(STANDARD)
D
PRECISION CAPACITOR
G R CAPACITOR - 75MMF
CARDWELL CAPACITOR - 2MMF
CRYSTAL
(EQUIVALENT CIRCUIT)

FIG. 1.

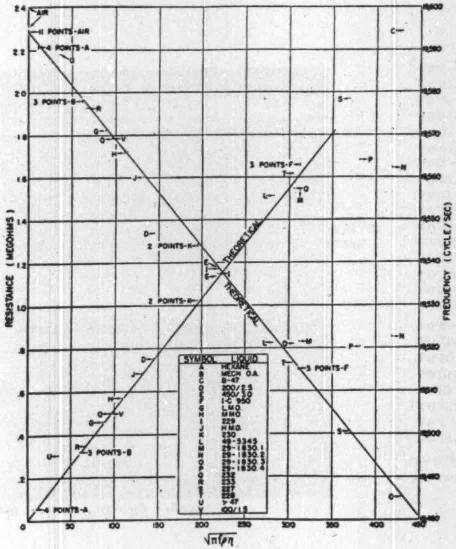
RESISTANCE (MEGOHMS)
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
.8
.6
.4
.2
0
FREQUENCY (CYCLE/SEC)
18600
18590
18580
18570
18560
18550
18540
18530
18520
18510
18500
18490
18480
0
50
100
150
200
250
300
350
400
450
$\sqrt{n/\rho\eta}$
AIR
11 POINTS-AIR
4 POINTS-A
3 POINTS-B
3 POINTS-F
2 POINTS-K
2 POINTS-K
3 POINTS-F
3 POINTS-B
4 POINTS-A
THEORETICAL
THEORETICAL
SYMBOL LIQUID
A HEXANE
B MECH O.A.
C B-47
D 200/2.5
E 450/3.0
F I-C 950
G L.M.O.
H M.M.O.
I 229
J H.M.O.
K 230
L 49-5345
M 29-1830.1
N 29-1830.2
O 29-1830.3
P 29-1830.4
Q 232
R 233
S 227
T 228
U Y-47
V 100/1.5

FIG. 2.

TABLE—*Reproducibility of Measurement in Low-Viscosity Liquids.*

Crystal	Liquid	$\sqrt{\pi f \rho \eta}$		Conductance (μ mho) Frequency (Cycles Per Second)									Maximum Deviations
A_2	Heptane...	11.90	Conductance.......	16.990	16.969	16.976	16.961	16.973	....	....	....	....	±0.17%
			Frequency.........	19,591.9	19,592.0	19,592.1	19,592.0	19,592.1	....	....	....	....	±0.1 ~
A_3	Heptane...	11.89	Conductance......	17.040	17.005	17.026	....	....	....	....	....	....	±0.1%
			Frequency.........	19,592.1	19,592.0	19,592.0	....	....	....	....	....	....	±0.05 ~
C	Hexane....	10.51	Conductance.......	17.296	17.314	17.314	17.315	....	....	....	....	....	±0.05%
			Frequency.........	19,590.6	19,590.6	19,591.1	19,591.0	....	....	....	....	....	±0.30 ~
C_1	Heptane...	11.90	Conductance.......	15.954	15.962	15.983	15.991	15.977	15.860	15.972	15.951	....	±0.41%
			Frequency.........	19,591.3	19,591.3	19,591.3	19,591.5	19,591.6	19,592.1	19,592.0	19,592.0	....	±0.4 ~
D	Hexane....	10.51	Conductance.......	18.295	18.518	18.357	....	....	....	....	....	....	±0.61%
			Frequency.........	19,594.4	19,594.4	19,594.3	....	....	....	....	....	....	±0.05 ~
	Heptane...	11.90	Conductance.......	16.718	16.840	17.046	....	....	....	....	....	....	±0.96%
			Frequency.........	19,594.0	19,594.1	19,594.3	....	....	....	....	....	....	±0.15 ~
E*	Heptane...	11.89	Conductance.......	16.059	16.713	16.132	16.419	16.165	16.150	16.145	16.191	16.839	±2.4%
			Frequency.........	19,585.5	19,586.3	19,586.3	19,586.4	19,586.3	19,586.3	19,586.3	19,586.3	19,586.4	±0.45 ~
F	Hexane....	10.51	Conductance.......	19.299	19.183	19.182	19.284	19.287	19.891	18.725	20.895	....	±5.7%
			Frequency.........	19,576.1	19,576.0	19,576.0	19,576.3	19,576.2	19,576.3	19,575.9	19,576.4	....	±0.25 ~
F_1	Hexane....	10.51	Conductance.......	18.690	18.686	....	....	....	....	....	....	....	±0.01%
			Frequency.........	19,571.1	19,571.5	....	....	....	....	....	....	....	±0.2 ~
F_2	Heptane...	11.90	Conductance.......	17.575	17.566	....	....	....	....	....	....	....	±0.03%
			Frequency.........	19,569.2	19,569.1	....	....	....	....	....	....	....	±0.05 ~
F*	Heptane...	11.90	Conductance.......	15.512	15.541	15.565	15.525	15.557	15.490	15.550	15.579	....	±0.29%
			Frequency.........	19,582.1	19,582.1	19,582.1	19,582.2	19,582.1	19,582.1	19,582.1	19,582.1	....	±0.05 ~
L	Hexane....	10.51	Conductance.......	17.083	17.081	17.128	17.136	17.124	17.135	16.931	16.926	....	±0.62%
			Frequency.........	19,690.1	19,690.5	19,690.5	19,690.5	19,690.5	19,690.5	19,690.3	19,690.3	....	±0.2 ~

weight oils. The points falling off the linear plot (*M, N, O, P*) were obtained in measurements of oils with polymeric additives. This latter behavior is typical of the response of polymer solutions.

The primary objective of the study was to improve the reproducibility of the torsion crystal system and most of the measurements were made using simple liquids. The table shows the reproducibility which was achieved after the method of mounting the crystal and the measuring equipment had reached a satisfactory degree of refinement. Good reproducibility of measurements depends almost entirely on the method of mounting the crystal. The crystal should be supported by phosphor bronze wires soldered to positions of minimum motion on the crystal surface. The best form of supporting lead is a phosphor bronze wire with cold-worked head which can be prepared by an upsetting operation in a suitable die. An assembled crystal is shown in Fig. 3.

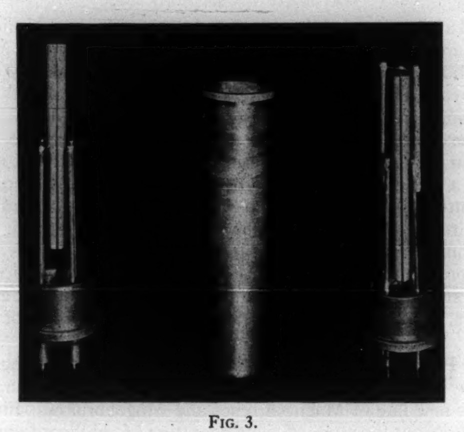

FIG. 3.

Aluminum Gears with Bonded-in Steel Hubs.—A new gear made of aluminum, with a bonded-in steel hub, has been developed by the Al-Fin Division, Farmingdale, L. I., of the Fairchild Engine and Airplane Corporation, according to M. V. Little, Acting General Manager of the Division. Lighter, and stronger than molded resin and fiber gears, the Al-Fin bonded gear can stand heavier loads.

Perfected after two years of research and testing, the gears, which are hobbed and shaved, are as quiet in operation as the composition type. Made of aluminum alloy, bonded to steel with a molecular process originally developed by Al-Fin, the gears have a tensile strength of about 15,000 psi. With steel hubs there is no danger of the aluminum gear conforming to the shaft. The gear has withstood shear tests of the bond up to 98,000 lb.

Aluminum alloys are used for strength and heat treatment of the casting and develop a Brinell hardness of 85 to 120, depending upon requirements.

The patented Al-Fin process of molecularly bonding aluminum and its alloys to steel and iron is a development which permits the fabrication of bi-metallic assemblies combining selected physical properties of both metals and facilitates the production of units requiring the strength, hardness, resilience, and fatigue resistance of steel with the light weight, high heat conductivity, excellent bearing properties, anticorrosive qualities, and other characteristics of aluminum.

Magnetic Particle Clutch.—The first commercially-usable magnetic-particle clutches have been introduced by the Vickers Electric Division of St. Louis, Mo., with its new line of Magneclutches and Magnebrakes. First developed for use by the Navy for more efficient operation of shipboard equipment such as radar, sonar, fire control and other devices, Vickers Magneclutches and Magnebrakes are now in production for use by industry for control of torque, speed and position.

The Vickers Magneclutch is a controllable coupling which utilizes the linking action of a dry magnetic mixture in a magnetic field between driving and driven parts to transmit torque. The magnetic mixture is composed of iron particles and flake graphite. The magnetic field is established by current flowing through a coil, and by varying the current the degree of clutching can be controlled.

Design and operating advantages offered by the new Vickers clutch that indicate widespread usage throughout industry include: small control power with extremely fast response, no wear on torque transmitting surfaces, torque at zero slip, large maximum to minimum torque ratio and easy adaptation to remote control.

Cancer Treatment Advanced by Radar Research.—Development of the first 4,000,000 electron volts linear acceleration high voltage equipment for cancer treatment and nuclear physics research was announced by Philips Industries during the International Radiological Congress held in London, July 24th to 30th. The equipment, developed by radar scientists, produces ultra-high penetrating X-rays and has a rating of plus or minus 250 Roentgens per minute at 40 in. distance. The first unit is now in operation in England with more units to be available by the end of the year. The new equipment is surprisingly compact, provides great flexibility in operation and is more economical than any equipment heretofore considered. The tube is accommodated on a Gantry type of stand.

NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION

RESEARCH RECONSTRUCTED

A REVIEW

Research and its Organization, by Dr. Ellice McDonald. A Brochure. Newark, Delaware, 1950.

Scientific research has often been considered to consist of a single mechanism, the scientific method, which, when applied properly and without prejudice, would yield automatically and, it seems, almost without effort significant results of either a positive or negative nature. The true scientist, moreover, has long been described as one who would conduct his experiments habitually within these limitations. No other criterion for research was ever considered necessary except, perhaps, financial backing. However, another mechanism, needful to present and future scientific research has always been present wherever research has been organized and directed, but its presence has been neglected and its importance unrecognized. *Research and its Organization* is, therefore, not only an important contribution to the scientific world, but also an equally important one philosophically, for in it is contained the description and analysis of a working model of a research organization of which Dr. Ellice McDonald, himself, is the director.

The brochure is in part a collection of articles which have appeared from time to time in this Journal; in part, a discussion of the interplay between biochemical and medical research written jointly by Dr. McDonald and Dr. William A. Mosher, chemical advisor to the Biochemical Research Foundation; and, in part, an index to both the publications of the Biochemical Research Foundation and, separately, the publications of Dr. McDonald, compiled by Mrs. A. H. Longenbach, librarian of the Foundation. Each of the parts has value in itself and has been published heretofore as such, but here the parts serve an additional purpose in indicating the correct approach to the organization of a cooperative research program with a single objective, the disclosure of biological secrets for the alleviation of human ills.

The hub of the research organization is the director. This must be honestly and unequivocally recognized not only by the director, himself, but also by the research workers, the advisors, and the trustees. One may imply from Dr. McDonald's analysis of the rôle of the director that he must meet, in addition to the qualifications outlined, those of the idealist and the philosopher—an idealist because he must constantly direct the efforts of the scientists at his command toward the ultimate revelation of biological secrets, a philosopher because he must have a sound knowledge of organizational structure and its function. The choice of research workers, the unification and correlation of seemingly independent phases of research in progress by research specialists such as organic and analytical chemists, cytologists, and physicists, and the publication of research work successfully completed and submitted all reflect the managerial abilities and the scientific bent of the director.

Research and its Organization contains, then, the model, the scheme for the building-up of a definite research program, philosophically conceived and scientifically instituted. The purpose and the goal of the Biochemical Research Foundation may be inferred from a joint study of the chapter on "Biochemical and Medical Research" and the index of the Foundation's publications. Here is shown how the idealism of the director finds achievement in the actual output of the research organization. The index of publications, too, substantiates the reliability of Dr. McDonald's methods of direction which are outlined in the "Choice of Research Projects," "Cooperation in Research," "Direction of Research," "Choice of Workers in Research," and "A Bonus System in a Research Organization." Confirmation of the thesis that the research organization must be fitted to the purpose and the goal of the research program envisioned is presented by Dr. Mosher, and belongs to the brochure as a corroboration of the techniques of directorship practiced at the Foundation for the past two decades. Unique, and essential, is the bibliography of Dr. McDonald, "the titles" of which "show the direction of interest over the years." This index shows the importance of the scientific leanings of the director; moreover, the subject matter, when compared with the present pattern of scientific thinking of the author, establishes the importance to the research organization of the director's own, personal, scientific ideal.

Research and its Organization presents to the scientific world, therefore, the pattern, the scheme, the philosophy, the mechanism, long neglected and much needed, of organizational and directional research.

EUGENE D. DAY

BOOK REVIEWS

CLASSICAL MECHANICS, by Herbert Goldstein. 399 pages, illustrations, 16 × 24 cm. Cambridge, Addison-Wesley Press, 1950. Price, $6.50.

This book presents a matured discussion of the essential theories of Classical Mechanics. Its title emphasizes a trend toward the division of mechanics into two major components; *viz.*, classical and quantum mechanics. In view of the considerable change from classical to quantum ideas, together with new aspects in atomic structure, and the demands of more mathematical technique in modern physics, the presentation of classical mechanics requires a new approach.

Along these lines the following may be noted. Vector and tensor notations are used throughout. The properties of the transformation matrix are discussed. The Lorentz transformation and special relativity with a careful presentation in the modification of the Lagrangian function are included as necessary in the curriculum. Emphasis, as it should be, is given to variational methods, together with a careful presentation of the Hamiltonian equation of motion. This is followed by a discussion of canonical and contact transformations, and the Hamilton-Jacobs theory, and action angle variables. This background, together with a discussion of geometrical optics, permits an effective presentation of the transition to wave mechanics and the Schrodinger wave equation.

The first part of the book is a recapitulation of basic elementary principles. The discussion of constraints and the elimination of the reactions of constraints through the equation of virtual work, leads to its formulation in generalized coordinates. While the equation of virtual work, in terms of generalized coordinates, is usually credited to Lagrange, the author maintains it as a generalization of D'Alembert. A welcome discussion (which assumes a knowledge of Maxwell's equation) shows the significance of the vector potential.

The discussion of the two-body central force problem is in line with the type of presentation needed in quantum problems. It resembles a similar presentation by Slater and Frank. It is uncommon to find the Virial theorem presented in works of mechanics. Likewise the scattering problems in a central field of force.

The author introduces the important subject of orthogonal transformations and matrix analysis in the kinematics of rigid bodies. The subject of infinitesimal rotations is likewise presented in tensor notation. The space and time changes of a vector as effected by the rotation of a frame, and the motion relative to the frame, lead to Euler's Equations, and the often discussed Coriolis force.

The last chapters are devoted to the theory of small oscillations, with the transition from discrete to continuous systems and finally the Hamiltonian formulation for continuous systems. The subject of orthogonal transformation in tensor notation and the significance of the Eigen-value equation with the principal axis transformation, is in accord with a necessary grounding in the transition to similar relations in quantum mechanics. The generalized Eigen-value approach then is extended to resonant frequencies and normal coordinates in classical mechanical systems.

Classical mechanics is built on the basic principles of momentum, energy, and virtual work, presented by a mathematical generalization. The great danger of brevity in a mathematical presentation is in the student overlooking the important details in the physical coordination of the mathematical relations. These can only be acquired by well chosen problems. Some criticism along these lines can be found in various parts of the book. For instance, emphasis through a vector derivation of the Coriolis force overlooks it as a special case, with rotating frames of the more general principle of angular momentum. Again the author expounds the theory of the center of mass through a translation coordinate with Lagrange's equations. The translation coordinate implies that the corresponding forces and reactions, as well as the potential, must be of an external nature. A more physical approach of gyroscopic principles and rotating frames would strengthen the student's dynamic instincts.

The book is particularly commendable, however, in its coordinating influence, from the mathematical aspect, between classical and quantum mechanics. It is a valuable contribution to the modernization of classical mechanics.

RUPEN EKSERGIAN

AN INTRODUCTION TO EXPERIMENTAL STRESS ANALYSIS, by George Hamor Lee. 319 pages, illustrations, 15 × 24 cm. New York, John Wiley & Sons, Inc., 1950. Price, $5.50.

In recent years, the increasing use of experimental stress analysis to determine the actual stresses in highly complicated and redundant structures made it inevitable that a book be written on the subject. It is fortunate that the book is as good as that prepared by Professor Lee of the Naval Postgraduate School.

An Introduction to Experimental Stress Analysis evolved from the lecture notes used by Professor Lee in his course in experimental stress analysis. These notes have undergone considerable revision since first used and the result is a fine, readable book on the whole field of experimental stress analysis. Although only an introduction, the book comprehensively surveys all the aspects of this comparatively new branch of mechanical engineering. Some aspects, of course, are treated in more detail than others, but all are mentioned. References for more advanced study follow each of the eleven chapters.

In the opening chapter, an unusually clear exposition of the fundamental concepts of the theory of elasticity is presented, including Mohr's circle, the differential equations of equilibrium and compatibility, Airy's stress function, and Saint Venant's principle and theory of torsion. In succeeding chapters, then, are discussed stress determinations from strain measurements; structural similitude including dimensionless analysis; the various mechanical, optical, acoustical and electrical strain gages; photoelasticity including the apparatus and experimental procedure; the membrane, hydrodynamic and electrical analogies for the solution of difficult problems in elasticity; the brittle lacquer method for the visual determination of strain; and such miscellaneous methods in experimental stress analysis as the brittle material method and the photogrid method. In the concluding chapter excellent suggestions concerning various experimental factors such as choice of method, interpretation of strain gage readings and calibrations, model loadings, and sources of errors are presented. The last chapter, also, contains an experimental problem in stress determination and several suggested laboratory exercises for the student to gain facility in experimental procedures.

An Introduction to Experimental Stress Analysis is soundly recommended for students just beginning experimental stress analysis as well as for engineers already familiar with the subject, as a valuable reference book.

E. W. HAMMER, JR.

AN INTRODUCTION TO PROBABILITY THEORY AND ITS APPLICATION, by William Feller. Vol. 1, 419 pages, illustrations, 16 × 24 cm. New York, John Wiley & Sons, Inc., 1950. Price, $6.00.

It is seldom that a mathematical subject like probability is treated in the most absorbing way in which it is treated in this new book by Prof. Feller. Probability theory is presented as a self-contained mathematical subject. The reader, especially one who is not a mathematician, cannot but feel the great power which is intrinsic to a probability approach to certain problems, especially when he sees the many practical applications set forth in the book.

The restriction is made early in the book that only discrete sample spaces will be considered, that is, an abstract space in which the probability measure is defined. While from the mathematician's point of view this restricts the field, there is so much that is included within the restrictions that there really is no restriction. The elementary topics in probability are followed by more advanced topics as the text progresses.

Following a short chapter on sample space and definitions, there are introduced the elements of combinatorial analysis and simple ordering and occupancy problems. The latter are illustrated by the Boise-Einstein and Fermi-Dirac statistics in a way which allows a firmer understanding of their meaning and nature than is usually presented in a formal course in modern physics. The chapters on conditional probability and repeated trials lead to the binomial, Poisson, and normal distribution functions, and the subject of random variables and laws of large numbers.

Included in the more advanced topics are chapters devoted to generating functions and recurrent events, random walk or "gambler's ruin" problem, Markov chains with constant transition probabilities, and simple time dependent stochastic processes.

Each of the seventeen chapters is written according to the same pattern. New theoretical ideas and theorems are introduced, proven where necessary, and then followed with numerous applications from many fields including sociology, medicine, biology, and games of chance. Each chapter closes with a series of problems which serve as additional illustrative text material. Answers to most of the 340 problems are included.

Aside from mathematicians, physicists and other scientists who encounter problems in modern communications theory, modern physics, and the probabilistic aspects of the biological sciences will find much reference material in this volume. Needless to say, the book can be used as a valuable reference work for professional and amateur gamblers.

S. CHARP

HETEROCYCLIC COMPOUNDS, edited by Robert C. Elderfield. Vol. 1, 703 pages, 15 × 24 cm. New York, John Wiley & Sons, Inc., 1950. Price, $11.00.

This is the first volume of a series planned to cover the entire field of heterocyclic chemistry. Organic chemists will be grateful to Dr. Elderfield and his associates for undertaking this monumental work. The editor explains that the series is not intended to be a compendium of the field but is rather intended to show primarily the chemical principles involved; however, other aspects such as industrial uses and physiological and pharmacological properties are included. Emphasis is placed on the fact that the most recent developments have been included in this book.

The chemistry of monocyclic heterocycles containing one oxygen, nitrogen, or sulfur atom in three, four, five, or six membered rings is presented in this initial edition. The contents are divided into nine chapters each carefully prepared by specialists in their particular field. The subjects covered include: (1) ethylene and trimethylene oxides; (2) ethyleneimine; (3) derivatives of azete; (4) furan; (5) thiophene; (6) pyrrole and its derivatives; (7) monocyclic pyrans, pyrones, thiapyrans, and thiapyrones; (8) pyridines; and (9) piperidines and partially hydrogenated pyridines.

One of the most impressive parts of the book is Harry S. Mosher's chapter on the chemistry of the pyridines. It provides an excellent coverage of the field and is replete with references. The great importance of penicillin and its structure led to the inclusion of the chapter on the derivatives of azete. Elderfield, in accord with his plans, did not include a thorough treatment of the alkaloids because they have been covered in many other texts. Some readers may feel that the chapter devoted to the chemistry of thiophene lacks the systemization of the others, but in this field sharp classification would lead to excessive specificity and tend to produce a mere tabulation of individual reactions. The provision of an excellent subject and formula index at the end of this volume will certainly facilitate its usage.

The author most certainly accomplishes his avowed purpose which is to present a detailed treatment of the heterocycles emphasizing chemical principles rather than tabulating physical properties and techniques of synthesis although where this information is pertinent it is included. The material is presented in a lucid, easily read manner, and the extensive use of structural and functional formulae, together with hundreds of references, adds greatly to the clarity of the book. Any group doing research work in heterocyclic chemistry will find the first volume a great asset and anxiously look forward to the publications of subsequent members of this series.

DONALD H. RUSSELL

STEAM TURBINES AND THEIR CYCLES, by J. Kenneth Salisbury. 645 pages, illustrations, 15 × 23 cm. New York, John Wiley & Sons, Inc., 1950. Price, $9.00.

Mr. Salisbury has been with the General Electric Company for many years as an engineer, in various capacities, concerned with sundry phases of steam turbines. His book, *Steam Turbines and Their Cycles*, fortunately reflects his wide, practical experience. Mr. Salisbury has written a practical treatise on steam turbines, explaining as simply as possible the various factors in steam turbine design. His opening chapter on thermodynamics typifies his approach to the various phases of steam turbine design; he states that the ignorance that embarasses most engineers concerning thermodynamics is largely due to lack of proper presentation, to "the introduction of too many ramifications" and "the use of too few clarifying examples." Throughout the book the ramifications of the various subjects are minimized and frequent examples presented to clarify the theory. For a subject as complex as steam turbine design, such an approach aids greatly in comprehension.

Steam Turbines and Their Cycles is divided into four parts: Fundamentals of Turbine Design, The Regenerative Cycle, Cycle Analysis, and Application Engineering. Part I commences with a chapter on thermodynamics, containing, however, only those fundamentals necessary to an understanding of steam turbines. Following, then, are chapters on power-plant cycles, the flow of fluids, a general description of turbines, turbine external characteristics, and steam path design. Part II contains chapters on regeneration and non-extraction performance, heat-balance practice, and a short cut to turbine-cycle heat rate. Part III has chapters on heater-cycle losses, cycle equations for homogeneous arrangements, arrangement and distribution losses, terminal difference and pressure-drop loss, evaporation of make-up and miscellaneous cycle corrections. In Part IV are chapters on design detail, estimating steam rates and heat rates. An appendix contains such valuable data as theoretical steam rates in pounds per kilowatt hour, large scale Mollier charts, properties of superheated steam, saturation-temperature and saturation-pressure tables, and selected references on steam turbines. In addition to the many illustrative examples, there are problems for the student at the end of each chapter.

The usual steam turbine design calculations are long and laborious. Mr. Salisbury has presented short cut methods to eliminate much of this lengthy computation, providing, for example, such labor-saving devices as curves for the design of a nozzle-bucket combination, charts for calculating partial-load steam rates, and approximations to the usual steam flow equations. Containing such a wealth of practical data, *Steam Turbines and Their Cycles* is, in addition to an excellent text, an excellent handbook, furnishing sufficient information to approximate quite accurately the design of a steam turbine.

E. W. HAMMER, JR.

PROPERTIES OF METALS AT ELEVATED TEMPERATURES, by George V. Smith. Edition 1, 401 pages, illustrations, 16 × 23 cm. New York, McGraw-Hill Book Co., 1950. Price, $7.00.

With the development of more efficient power production and advances in the oil and chemical industries, increasingly higher temperatures of operation are required. Such operations demand materials that will withstand the effects of high temperature. Consequently, it is imperative to know the reactions of metals at elevated temperatures.

Generally, the physical properties of metals are not so good at abnormal as at normal temperatures. While the knowledge of the metallurgy of metals at elevated temperatures is far from complete, Dr. Smith, in *Properties of Metals at Elevated Temperatures*, performs a valuable service to the metallurgical and design engineer by compiling all available information and correlating the research results of the past twenty-five years on plastic deformation at high temperatures, effect of grain size, recrystallization and phase changes with the use of metals at elevated temperatures. Recognizing that information is scarce, Dr. Smith frequently indicates where additional research, or review of existing data, is necessary to bridge the numerous gaps in the knowledge.

Briefly, plastic deformation, creep and fracture of single crystals and polycrystalline metals, and theories of flow and fracture are discussed in the first six chapters. The nature and means of physical testing for the above characteristics are then described in a chapter on tests and test equipment. The effect of chemical composition, melting and deoxidation practices, heat treatment, grain size, varying temperature, the state of stress, and varying stress on creep are treated in three succeeding chapters. Scaling and microstructural changes during service at high temperatures are discussed in the eleventh chapter. In the concluding chapter the author briefly but ably surveys the theoretical and practical considerations in the problem of adequately designing for elevated-temperature service. When reading this book, it might be well to scan the last chapter first, to obtain the general view of the problem of design of engineering structures and the various factors that must be considered.

The numerous charts and graphs and the almost five hundred references add greatly to the value of the book.

E. W. HAMMER, JR.

POCKET ENCYCLOPEDIA OF ATOMIC ENERGY, by Frank Gaynor. 204 pages, drawings and tables, 14 × 22 cm. New York, Philosophical Library Inc., 1950. Price, $7.50.

This book presents a convenient alphabetical catalog of terms and explanations related to the fields of Atomic and Nuclear Physics. It is neither a popular treatment nor one destined for the specialist, for it contains both readable descriptions, such as might be useful to a librarian, together with equations of nuclear processes understandable to a physicist.

Sketches including for example the uranium reactor and the synchrotron make the book more than a simple dictionary. Likewise, the more important general subjects are discussed at some length and referenced to related material. Numerous tables having to do with elementary materials and their physical structures have been collected.

The author has produced a volume which is as up-to-date as security will permit. Evidence of this is contained in a several-page discussion on the subject of the "hydrogen" bomb. However, a few terms from other fields which have little specifically to do with Atomic Energy could yield their space to better advantage. For example, electrical terms such as "henry" and "vacuum tube" do not have direct application and are too general to have been included.

In any field which is new and expanding rapidly the author must realize his work is sharply dated, and many subsequent editions will be needed to keep this particular volume in demand. This is especially true of an encyclopedia where the text ventures beyond the bare, fundamental facts. It is suggested to the publisher, therefore, that a less formal presentation taking the form of a paper-cover volume reproduced in off-set might yield a more economical and practical form of the book which would enjoy wider distribution of successive editions.

C. W. HARGENS

PHOTOELASTICITY, PRINCIPLES AND METHODS, by H. T. Jessop and F. C. Harris. 184 pages, illustrations, 16 × 24 cm. New York, Dover Publications, 1950. Price, $3.50.

The authors, both associated with the University College, London, have written a book intended for those interested in practical photoelastic stress determination. Mathematics to the extent covered in undergraduate technical work is adequate to follow the derivations and examples cited. The book is well suited for the engineer who wishes to use photoelasticity as a tool without delving into lengthy theoretical texts on photoelasticity.

The treatment of stress and methods of describing stresses in a plane stress field are similar to the presentations found in books on strength of materials. Similarly, the description of strain and stress-strain relations is of an elementary nature.

A section on optics and a description of polarization and double refraction are introductory to a simple explanation of the theory of photoelasticity adequate for the engineer who intends to use photoelasticity as a practical tool. A section on interpretation of observations describes the meaning of various configurations encountered in isochromatic and isoclinic patterns. Methods are given for determining the separate principle stresses by measurement and graphical analysis. A short section is given to stresses in three dimensions and small mention is made of the "frozen-stress" method used in three dimensional photoelasticity.

A section on the photoelastic bench gives diagrams of optical systems and discusses various optical set-ups for specific photoelastic purposes. Components for the optical bench and loading frame are described. A section on materials and procedure points out advantages and disadvantages of various photoelastic materials and the precautions to take when preparing models from these materials. Techniques for recording isochromatics and isoclinics are discussed.

The book is concluded with a section on practical problems that have been solved photoelastically. The examples given have been pretty well covered in American literature. A short appendix treats certain stress and optical considerations in more detailed manner than in the text.

R. H. MARVIN

METHUEN'S MONOGRAPHS ON PHYSICAL SUBJECTS. Illustrations, 11 × 17 cm. London, Methuen & Co. Distributed in U. S. by John Wiley & Sons, Inc., New York.

This series is intended to supply readers of average scientific attainment with a compact statement of the modern position in each subject. Eleven titles are here briefly reviewed. Others in the series were reviewed in this JOURNAL for June, 1950, p. 507.

Collision Processes in Gases, by F. L. Arnot. 104 pages. 1950. Price, $1.25.

A very clear, simply written account of the observations and theory of collisions between electrons, photons, etc., and nuclei in gases at very low pressures is given. It is well written as an introduction to the field, but unfortunately contains no material more recent than 1933. Included are chapters on critical potentials, both ionization and excitation, and their measurement. The calculation and determination of the probabilities of the occurrence of these processes is discussed, together with the concept of collision cross-section of the atom, and the observed angular distribution of scattered electrons. Under the heading of other collision processes, light scattering, photo excitation effects, and collisions between normal and excited atoms are discussed. A bibliography of general and text references, all prior to 1933, is given.

A. D. FRANKLIN

Wave Mechanics, by H. T. Flint. 131 pages. 1949. Price, $1.25.

Many varied phenomena come under the heading of waves—sound in organ pipes, vibrations in strings, light, and even the probability that an electron is in the middle of a box. All of these have at least one thing in common—the mathematical quantity by which we describe the state of the system (pressure in a sound wave, field strength of light, eigenfunction of the electron) is related to the variables of space and time through the same general equation, the wave equation. A discussion of this equation, its properties and solutions, and its applications, is here given. Because the content is primarily of a mathematical nature, the fact that it is essentially a book of the mid-thirties is no drawback. It is an interesting approach, and it is a valuable little book.

A. D. FRANKLIN

An Introduction to Vector Analysis, by B. Hague. 122 pages. 1950. Price, $1.25.

This monograph is based on a series of lectures given by the author at the Polytechnic Institute of Brooklyn. The material is intended to give a working knowledge of vector analysis to engineers and students of physics so that they may use the principles involved in the solution of physical problems. Matters which are of an exclusively mathematical interest are treated only briefly.

The author includes a bibliography listing suitable books for those who wish to pursue the subject in more detail, as well as several other important books which treat physical subjects by vector methods. The book is clearly written and fulfils quite well the purpose for which it was written.

M. D. EARLE

Wave Filters, by L. C. Jackson. 107 pages. 1950. Price, $1.25.

This monograph was written primarily for students of physics and radio who wish to learn the fundamental principles involved in the design of wave filters. The author develops the mathematical theory in considerable detail. The most important topics covered are: types of wave filters and their applications, the design and construction of wave filters and the application of filter theory to mechanical and acoustical problems.

The monograph also contains a bibliography listing material for those interested in further study of the subject.

M. D. EARLE

X-Ray Crystallography, by R. W. James. 88 pages. 1950. Price, $1.25.

In this monograph is found a very good introduction to the basic concepts of crystal structure as derived geometrically, the action of the crystal lattice as a diffraction grating for X-rays, and the determination of crystal structure from X-ray diffraction measurements. The final chapter of the book is devoted to a discussion of some of the principles which appear to govern the formation of crystals, as indicated by the findings obtained thus far in X-ray studies.

Mr. James' book provides one with a good general picture of the method of attack which must be followed in attempting to deduce the crystal structure of a given material from the X-ray data obtained. For a more detailed account of both the theoretical background and the calculation procedures involved, the interested reader can refer to the texts recommended in the bibliography at the end of the book.

JEAN A. MINKIN

Atomic Spectra, by R. C. Johnson. 120 pages. 1950. Price, $1.25.

This monograph treats in a brief and comprehensive manner the most important topics involved in the field of atomic spectra. It describes spectra of the hydrogen-type and the more complicated spectra of atoms with two or more valence electrons. Other subjects discussed are: the structure of atoms, the fine structure of spectral lines, the periodic table and the effect of electric and magnetic fields on spectral lines.

The material is described in terms of concepts that can be easily visualized by students of physics. The use of the new quantum theory is almost completely avoided due to its highly mathematical nature.

M. D. EARLE

Heaviside's Electric Circuit Theory, by H. J. Josephs. 115 pages. 1950. Price, $1.25.

This compact volume is devoted to the theme that the best method of attacking the more difficult problems in electric circuit theory is the direct operational method of Heaviside. It endeavors to demonstrate the application of Heaviside's operational calculus to electric circuit problems and to compare the operational equations for certain specific problems with explicit solutions obtained by classic methods. The author relates instances in which the classic methods fail and discusses certain theorems arrived at deductively by Heaviside and others such as Borel.

The application of these ideas to illustrative examples such as ladder networks and transmission lines touches briefly on the practical side, but the volume serves primarily as an interesting historical summary embodying the evolution of operational methods.

C. W. HARGENS

Kinetic Theory of Gases, by M. Knudsen. 64 pages. 1950. Price, $1.25.

This book is essentially an account of the author's own researches in the field of the kinetic theory. Such of the theory as is necessary to make the various experiments understandable is given, the experiments are described, and the results as they bear upon the kinetic theory are discussed. The author was particularly interested in phenomena observed at very low pressures, and so the discussion includes effusion, flow of gases through tubes, reflection of molecules from surfaces, evaporation and condensation, heat conduction, etc. The work was all done before 1933, and the book is of historical interest only.

A. D. FRANKLIN

Atmospheric Turbulence, by O. G. Sutton. 107 pages. 1949. Price, $1.50.

In this monograph the author treats the subject of turbulence in the lower atmosphere from a mathematical standpoint. He avoids the treatment of large scale motions of the atmosphere and makes no effort to describe the instruments used to obtain the data necessary for making specific calculations.

The book will probably be of most value to those interested in the theoretical aspects of dynamical meteorology.

M. D. EARLE

Applications of Interferometry, by W. E. Williams. 104 pages. 1950. Price, $1.25.

Here is one of the Methuen series of monographs that definitely suffers because of its age. It describes in good detail the types and uses of interferometers. The differences between types using point or line sources, and extended sources, are discussed as well as diffraction gratings, multiple beam instruments, interference fringes, etc. Attention is confined solely to the instruments and their uses. Since, however, the material all dates from before about 1930, the book is now of interest mostly as an historical collection.

A. D. FRANKLIN

X-Rays, by B. L. Worsnop and F. C. Chalklin. 126 pages. 1950. Price, $1.25.

One of a series of monographs intended to provide for a wide field of readers a compact résumé of the recent state of development in various branches of physics, "X-Rays" contains a brief but informative description of the methods of production and observation of X-rays, the determination of their wavelengths from crystal diffraction studies, X-ray emission spectra, absorption, scattering of X-rays by various types of materials, optical phenomena observed, and photoelectric and ionization effects produced by X-ray bombardment.

Such an amount of material is of course impossible to cover in great detail in a book of this length (126 pages). However, in conjunction with each chapter the authors present in their bibliography adequate references for the reader who wishes to pursue any given topic further.

As an introduction for the physics student interested in X-rays and for all others seeking a good general idea of the present-day situation in this science, this book is recommended reading.

JEAN A. MINKIN

CURRENT TOPICS

"Mechanical Brain" for Structural Computations.—An apparatus which computes the stresses in members of bridges, roof trusses and ship frames mechanically, and does so in many cases where the human mind has been found inadequate, has been developed. Small models of the structures or frames are made up of wires, tubes or bars which connect into standard joints, loads are applied by spring scales, and the stresses in the component members or the bending moments at the ends of the members are measured directly in pounds or inch pounds.

The device, known as the stressometer, and invented by M. Berman, president of the New York Society of Engineers, works on a simple mechanical principle—namely, that if the end of a member under stress is released slightly so as to reduce its stress, it will take as much force to get the end back to original position as there was force in the member originally. For example, if it is desired to find the stress in a model member stressed to 7 lb. by the spring scale loads, the end of the member is disconnected, and brought back to position with a special spring scale known as a "test scale," which test scale will have to exert 7 lb. force to get the member end back to position, as seen on a gage in circuit.

This is the method for highly complicated "statically indeterminate" structures which do not lend themselves readily to mathematical computation. But for simpler "statically determinate" structures a special stress unit may be attached and made a part of the member itself, and the stress read on that stress unit when loads are applied to the model. Thus, if a stress unit consisting of a pair of stiff, flat springs is inserted within a model member the springs will deflect slightly when the member is stressed, and this deflection is read in terms of pounds on a gage. This method cannot be used for statically indeterminate structures because in such the member must remain its original length at the time of measurement; thus, in the measuring method first described, the member will be its original length at the time the end of the member is back to original position and the stress reading made.

The second method described—for measuring determinate stresses—the most usual conditions for beams, trusses and frames, is readily incorporated in an inexpensive instrument for use of physics, mechanics and structural engineering students in verifying the principles of statics, parallelogram of forces and the methods of mathematical computation now used; at the present time no adequate devices to illustrate these basic principles are made, and it is hoped that engineering education will advance by the use of the instrument.

Some problems which may be solved with the stressometer are obtaining beam reactions, stresses in members of trusses, bending moments in rigid frames, stresses in three-dimensional structures, secondary stress moments, continuous beam reactions and moments, influence lines for both stress and moments for moving loads, the principle of equilibrium of moments, horizontal and vertical forces on a structure and around a joint, and miscellaneous structural problems. The instrument measures forces but not variations of unit stress, for which other laboratory methods may be used.

Projection Magnifier.—A device which magnifies the print in books, magazines and newspapers to five times original size for the benefit of partially sighted people has been developed at The Franklin Institute Laboratories for Research and Development after several years of research.

The new instrument, known as a "Projection Magnifier," was developed under a grant from the W. K. Kellogg Foundation, and is now in the final testing stage. Weighing a trifle more than 17 lb., the Magnifier is easily portable and may be used wherever there is a convenient electrical outlet. It measures 12 in. high, 14 in. wide and 27 in. deep and contains two ordinary 40-watt bulbs.

With this machine many persons having as little as 20/200 corrected vision, considered legally to constitute blindness, can read average type. When the page is placed on a movable tray at the bottom of the machine and a lens lowered against it, an illuminated magnification appears on ground glass slanted at an angle correct for the reader sitting comfortably in a chair before the machine.

Several such devices are now being *used experimentally* in libraries and at institutions for the blind. Wallace Frank, Franklin Institute engineer in charge of the development of this device, demonstrated it at the 1950 convention of the Association of Instructors for the Blind. He stated that the machine is not only compact and easily handled but also has little on it to get out of adjustment and cost of upkeep is negligible.

Rocket-propelled anchors may solve landing problems in confined or rough terrain, when perfected by the Air Materiel Command, Dayton, Ohio. Consisting of a 28-in. steel tube $1\frac{3}{8}$ in. in diameter, filled with rocket propellant, the anchor may be fired 18 in. into the ground at a 45-degree angle by a glider or small plane settling to the ground. Shock is absorbed by 200 ft. of stainless-steel tape wound on a hydraulic brake.

Warfarin.—A new rat and mouse killer—hailed as one of the safest from the human standpoint and one which holds promise of wiping out rodent colonies completely and permanently—is now available to the general public, it has been announced by the Wisconsin Alumni Research foundation (WARF).

Warfarin, the new death-dealing substance, developed at the University of Wisconsin in the laboratories of Prof. Karl Paul Link, is now available to rodenticide manufacturers, reported Ward Ross, WARF general manager.

Warfarin's effectiveness will make it of great value for use on farms and in restaurants, warehouses, and manufacturing establishments. Other poisons kill so rapidly that dead and dying rats serve as a warning to the rest of the colony. Tasteless and odorless Warfarin, however, is slow-acting and relies upon the cumulative effects of small quantities consumed over a period of days.

Poisoned rats show no immediate effects. They eventually become drowsy, and begin to walk with a slow and measured gait. Finally, without suffering, they die of internal hemorrhage. Other rats are not warned of the poison—there is no dash for water, there are no convulsions—and the bait then remains to keep stray rats from again populating the area.

One major advantage of Warfarin rodenticides is their relative safety. Since they relay upon cumulative doses for results, there is little chance that enough bait will be taken by humans, pets, or domestic animals to cause adverse effects. Out of the several hundred nation-wide tests made on Warfarin, no case of accidental human poisoning was reported.

Dwarf Galaxies May Throw Light on Evolution of Star Systems.—The National Geographic Society–Palomar Observatory Sky Survey recently announced the discovery of two new dwarf stellar systems in the Constellation of Leo, one of which is the smallest ever found. The announcement was made in a paper by R. G. Harrington and Dr. Albert G. Wilson which was read at a meeting of the Astronomical Society of the Pacific in Salt Lake City, Utah.

The new systems are additions to what is known as our "local" group of galaxies and bring to 16 the number now included in the group. The so-called "local" group is one of which our own galaxy, or Milky Way, is a member.

Sky Survey plates made in April of this year (1950) by the 48-in. Schmidt camera at the Palomar Observatory revealed the two new stellar systems.

The discoveries are significant because of the fact that further study of these new systems of stars may throw new light on the stability and perhaps the evolution of nebulae. This is particularly true of the one which is smaller than any heretofore known, since its discovery indicates that there is reason to assume that eventually galaxies of even smaller mass and size may be found. If this proves true, then there may be new evidence gained as to how huge island universes like our own Milky Way evolved. Heretofore the smallest known galaxies had a diameter of about 3,000 light years as compared to 1,500 light years for the smallest of the new ones.

It will be another year before additional work can be done on these new systems since they are now in a poor position for observation. However, when they are again in good position, large telescopes like the 200-in. Hale at Palomar and the 100-in. at Mt. Wilson will undoubtedly be turned on them in search of more detailed information.

It will then be possible to obtain more complete data on their velocity, luminosity and mass, since they are close enough to be resolved into individual stars. It is out of this added information, which might require several years to accumulate and evaluate, that astronomers may get new clues as to the stability and evolution of all stellar systems.

Development of 12 Per Cent Chrome Cast Steel.—Development of 12 per cent chromium alloy steel castings for use in gas-condensate wells in many parts of the world is helping to solve a multi-million dollar corrosion problem for the oil and gas industry, it was reported recently by Warden F. Wilson, general sales manager of Lebanon Steel Foundry.

Perfection of the corrosion-resistant cast steel alloy now being used in the production of valve bodies, fittings and similar parts required to withstand the terrific service pressures and corrosion factors involved, he said, is the outgrowth of intensive cooperative research extending over a period of more than seven years. Adoption of the 12 per cent chromium alloy, he added, has meant literal savings of millions of dollars in equipment life, operating safety, and minimization of costly service failures in the field.

Prior to development of the selected chromium cast steel, Mr. Wilson told a sales-engineering forum on pressure-corrosion products, vicious corrosive conditions in certain gas-condensate wells were found to destroy vital control parts of well-head valve (christmas-tree) equipment in as little as 30 to 90 days of service, with resultant heavy replacement costs, shut-down losses and actual peril to life and property.

Subsequent installations, utilizing the perfected 12 per cent chromium cast steel parts, it was explained, arrested the accelerated corrosion and provided an effective deterrent to the severe pitting and edge attack of gases and acids encountered at high pressures and velocities in unusual conditions of turbulence.

Tracing repeated tests made with 32 selected metals and alloys, Mr. Wilson cited numerous examples in which 12 per cent chromium steel parts, replacing old-type carbon steel fittings, have established perfect service records after more than three years of exacting use, whereas original carbon steel parts failed completely in a matter of weeks or months.

The alloy developed by the company, known as Circle-L 12, possesses physical properties sufficient to withstand the high internal pressures and test pressures of gas condensate well service often as high as 10,000 to 15,000 psi., with yield points ranging from 50,000 to 160,000 psi., depending on heat treatment, and a wide range of necessary mechanical properties.

To date, L-12 cast fittings have fulfilled the required performance of corrosion resistance without known exception, Mr. Wilson said, and adaptation of the chrome alloy steel as a standardized material, made under exacting specification requirements, is resulting in use of cast parts in many parts of the world.

Tachometer Generator Designed for Hazardous Locations.—The new Type E tachometer generator developed by Weston Electrical Instrument Corp. has been specifically designed for use where there may be explosive atmospheres. Available as either an a-c. or a d-c. generator, this type is listed by Underwriters' Laboratories, Inc., for use in two classes of hazardous locations: Class I, Group D (flammable and explosive mixtures of vapors and gases with air); and Class II, Groups F and G (combustible mineral and grain dusts in air). Its construction makes it particularly suitable for use in oil refineries, chemical plants, grain elevators, flour mills, coal mines and similar locations where explosive atmospheres may occur.

The Weston Type E tachometer generator is enclosed in a sturdy cast iron housing having four feet for mounting. The housing measures approximately 10¼ in. over-all and 4½ in. in diameter. Its ¾ in. diameter stainless steel shaft, with a machined flat for a gear or coupling, extends 1½ in. beyond the bearing support. Electrical connection is made in a terminal box tapped to take standard ½-in. conduit.

Both a-c. and d-c. generators are available for use in the new housing. The a-c. generator, Model 758, is of the rotating magnet type and is suitable for speeds up to 5000 rpm. Higher speeds can be accommodated on special order. The generator can be operated in either direction, but the associated indicator will always read in one direction.

The Model 750 d-c. generator has a wound armature with commutator and brushes. It can be operated at speeds up to 2000 rpm. in either direction. The associated indicator can be adjusted to read in either direction.